Mayan Mathematics

Patrick Brown

History of Math

Professor Van Opstall
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The Mayan civilization began around 2000 BCE in the southern region of what is now Mexico. At this time what is currently thought of as the Mayan civilization established its first settlements. These started to become more complex with the construction of burial mounds, the precursor to the well known stepped pyramids. Mayan culture continued to evolve until the classic period around 250 CE to 900 CE. By this time, the Mayans had expanded to Guatemala, Belize, and Honduras. The peak of the Mayan Culture was in 900 CE.

This advanced culture was first introduced to the European culture when Hernán Cortes arrived in the Yucatán peninsula with 11 ships, 506 soldiers, 100 sailors and 10 horses. Cortes had left Spain in 1505 after hearing stories of what Columbus had discovered in the New World. He first landed in Hispaniola which is now called Santo Domingo and farmed there until he left for Cuba with Velázquez in 1511. He remained there for some time before finally leaving for the Yucatán peninsula on February 18, 1519. The inhabitants Cortes met were the descendants of the Mayan culture, and they offered little resistance to Cortes' occupation.

Cortes' takeover opened the door for the next person who finally brought the mathematics of the Mayans to present society. Diego de Landa joined the Franciscan order in 1541 when he was only seventeen years old. He requested to be sent to the New World to be a missionary. It was here that he helped the Mayan People of the Yucatán peninsular and protected them from their Spanish occupiers. While doing this he learned about the Mayan culture and its long history. As he learned about the religious practices and the glyphs of the Mayan culture, he began to think that they were related to the devil. This might have been one reason why he ordered all Mayan idols destroyed and their old books burned. Interestingly, perhaps coming out of de Landa's regret for all the history he had destroyed, he then wrote a book *Relación de las Cosas de Yucatán* (1566). This book describes many aspects of Mayan culture like hieroglyphics, customs, temples, religious practices and history. This work was considered lost for many years but was then rediscovered in Madrid in 1869.

Some few artifacts survived de Landa's destruction, including the Madrid Codex which is in the American Museum in Madrid and the Paris Codex at the Bibliotheque nationale in Paris. The most important of these artifacts was the Dresden Codex which is kept in the Sachsische Landesbibliothek in Dresden. It is mainly concerned with astronomy, but the only surviving copy we currently have is believed to be transcribed in the eleventh century CE from the original source thought to be written between the seventh and eighth centuries CE. These artifacts do not, however, describe the methods the Mayans used to calculate only the results of these calculations-often surprisingly accurate.

The basis of the mathematics of the Mayans is their number system. The Mayan system was extremely advanced for its time. It consisted of a place value system and a cipher system combined together. One interesting aspect of the Mayan number system was that it was for the most part base twenty. One theory is that this is because the Mayan people were counting on thier fingers and toes for a total of twenty. This is supported by the fact that one of the words for twenty was "uinic" which translates to "man", seemingly meaning the total of his digits. Two other words were used for twenty, "kal" and "may". The system was not fully base twenty because instead of a 400's place there was a 360's place. This is presumably so it is closer to the length of a year. It has been speculated that the common people used a number system that was completely in base twenty while the priesthood used the aforementioned number system. However no record of it exists.

There were two basic units of the Mayan number system. The first was a dot used to represent the unit. The next was a horizontal line used to represent five. These were grouped to represent numbers up to twenty with up to three lines on the bottom and then up to four dots above the lines. There was one more symbol in their number system and that was a small shell glyph used to represent zero. This was very unusual in the number systems of ancient civilizations but it was very useful in their place value system. The Mayans also had a less commonly used symbol. This was used for numbers between 20 and 39 such as the number of days between two dates. The symbol was a moon used to represent twenty. Then a normal number up to 20 is placed by the moon and these are totaled to add up to 39. These moon hieroglyphics appear predominantly in the Dresden Codex. The arrangement was very interesting in that numbers were written vertically with the highest value place at the top, and zeroes where appropriate.

Another fascinating aspect of the Mayan number system was the appearance of what seems to be negative numbers. These so called ring numbers are numbers with the unit place surrounded by a red loop tied with a knot at the top. These are found in the Dresden Codex. They are usually accompanied by other numbers called companion numbers. It seems that these numbers are a number of days before the date that often accompanies these ring numbers.

The most common place that the Mayan number system was used was in the calendar and the calculations associated with it. The Mayan civilization actually used two separate calendars which would form the basis of a lot of their mathematical problems.

The first calendar called the "Haab" was a civil calendar somewhat similar to our current calendar. It consisted of eighteen months each with twenty days. There was then an additional period of five days called "Wayeb", which was considered unlucky. During this period Mayans would do very little. With this extra month the civil calendar was 365 days long.

The second calendar called "Tzolkin" was the ritual calendar. This calendar consisted of thirteen months again with twenty days each. It is theorized that these thirteen month come from the thirteen Mayan gods of the upper world. The twenty days in a month again seem to come from twenty being the number of fingers and toes a human has.

These two calendars operated independently thus creating a huge cycle. We can calculate the time it will take for the two calendars to align as

$$lcm(260, 365) = 18980$$

This is the total number of days it will take for the calendars to align. If we divide by 365 we see that it will take 52 years. Another important cycle to the Mayans was Venus. It has a Synodic period of 584 days. This aligns with the two other cycles in

$$lcm(18980, 584) = 37960$$

This would be two cycles of 52 years. This event occurring every 104 years would be a huge celebration in Mayan culture.

There was another system that the Mayans used to represent dates. This was an absolute timescale which is generally referred to as the "Long Count". This was based on a creation date which is predominantly cited as 12 August 3113 BCE, though some sources disagree. A Long Count date is then simply a number of days from this creation date. These numbers would be represented as days which have unique names for the place values. Days were referred to as kins. Then 20 kins is 1 uinal or month. 18 uinals are 1 tun which would be a year without Wayeb. After a tun the numbers return to base twenty and 20 tuns is 1 katun and 20 katuns is 1 baktun.

There were two major mathematical problems that the Mayans considered. The first was if one were given a date and a number of days from that date what would the new date be. The dates were referred to as triples (t,v,y) where t is the number of the month of the ritual calendar one to thirteen, v is the day of the ritual calendar one to twenty, and y is the day of the year one to 365. So given a date (t_0,v_0,y_0) and then n_5 baktuns, n_4 katuns, n_3 tuns, n_2 uinals and n_1 kins forward the date in the same (t,v,y) format is

$$
\begin{aligned}
t &= t_0 - n_5 - 2n_4 - 4n_3 + 7n_2 + n_1, \bmod 13 \\
v &= v_0 + n_1, \bmod 20 \\
y &= y_0 + 190n_5 - 100n_4 - 5n_3 + 20n_2 + n_1, \bmod 365
\end{aligned}
$$

The other major calculation that came up often in Mayan civilization was finding the minimum distance from two dates. Given two dates (t_1,v_1,y_1) and (t_2,v_2,y_2) we define the day in the ritual calendar ΔSR, the year number in the cycle of 52 as n_y and the day in the whole cycle of both calendars as ΔCR. Then we also define:

$$\Delta t = t_1 - t_2, \bmod 13$$
$$\Delta v = v_1 - v_2, \bmod 13$$
$$\Delta y = y_1 - y_2, \bmod 365$$

We can then get the shortest number of days between these two dates:

$$\Delta SR = 40\Delta t - 39\Delta v, \bmod 260$$
$$n_y = \Delta SR - \Delta y, \bmod 52$$
$$\Delta CR = 265 n_y + \Delta y, \bmod 18980$$

The Mayans also made other calculations. The first of these was the actual length of a year. The Mayans calculated this to be 365.242 days and our modern estimate is 365.242198 days. Then the Mayans calculated the length of a lunar month. At copán the Mayan's calculated that 149 lunar months where 4400 days which gives 29.5302 days as the length of a lunar month. Then at Palenque in Tabasco it was calculated that 81 lunar months was 2392 days which is 29.5308 days in a lunar month. Both of these are very close to the modern estimates of 29.53059 days in a lunar month.

Bibliography

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