

Name: _____ Student ID: _____

Quiz 1

Directions: You will have 30 minutes to complete this quiz. Please show all of your work and mark your answers clearly. You may not use any extra resources during the quiz: not your notes, not your book, not a cell phone, not a calculator. Good luck.

1. (6 points) Find the general anti-derivative $\int f(x) dx$ when

$$f(x) = \frac{1}{2\sqrt{x}} + 2 \sec(x) \tan(x) - 4 \cos(x) - e^{-3x}.$$

Solution: We find the general anti-derivative

1 pt for using linearity of Integral

$$\begin{aligned} & \int \left(\frac{1}{2\sqrt{x}} + 2 \sec(x) \tan(x) - 4 \cos(x) - e^{-3x} \right) dx \\ &= \int \frac{1}{2\sqrt{x}} dx + 2 \int \sec(x) \tan(x) dx - 4 \int \cos(x) dx - \int e^{-3x} dx \\ &= \sqrt{x} + 2 \sec(x) - 4 \sin(x) + \frac{e^{-3x}}{3} + C \end{aligned}$$

1 pt for each antiderivative calculated correctly

1 pt for remembering the "+C"

2. (6 points) A bullet is fired at time $t = 1$ with an initial velocity of $v = 400$. If the bullet's acceleration is given by the function $a(t) = -100t$, find the velocity of the bullet as a function of time.

Solution: We know that velocity must be an antiderivative of acceleration. Therefore we begin by finding the general antiderivative of the acceleration.

$$\int a(t) dt = \int (-100t) dt = -50t^2 + C.$$

2 pts for correct anti-derivative

Now we use the given initial condition to determine the unknown constant C .

$$v(1) = -50(1^2) + C = 400. \implies C = 450.$$

3 pts for substituting in initial condition & finding constant

This fully defines the velocity as a function of time.

$$v(t) = -50t^2 + 450.$$

1 pt. for writing correct function.

3. (8 points) From the book, we know that

$$\int_0^b x \, dx = \frac{b^2}{2}, \quad \text{and} \quad \int_0^b x^2 \, dx = \frac{b^3}{3}.$$

Use these facts and the properties of definite integrals to calculate

$$\int_1^2 (6x + 3x^2) \, dx.$$

(Do not use the Fundamental Theorem of Calculus, or the Evaluation Theorem)
Solution:

$$\begin{aligned} \int_1^2 (6x + 3x^2) \, dx &= \int_1^2 6x \, dx + \int_1^2 3x^2 \, dx && \text{(Rule 4: Sum \& Difference)} \\ &= 6 \int_1^2 x \, dx + 3 \int_1^2 x^2 \, dx && \text{(Rule 3: Constant Multiple)} \\ &= 6 \left(\int_0^2 x \, dx - \int_0^1 x \, dx \right) + 3 \left(\int_0^2 x^2 \, dx - \int_0^1 x^2 \, dx \right) && \text{(Rule 5: Additivity)} \\ &= 6 \left(\frac{2^2}{2} - \frac{1^2}{2} \right) + 3 \left(\frac{2^3}{3} - \frac{1^3}{3} \right) && \text{(From Problem Statement)} \\ &= 6 \left(2 - \frac{1}{2} \right) + 3 \left(\frac{8}{3} - \frac{1}{3} \right) \\ &= 12 - 3 + 8 - 1 \\ &= 16 \end{aligned}$$

Handwritten annotations:

- 4 pts for using linearity of Integral* (bracketed next to the first two lines)
- 2 pts for breaking up Interval properly* (bracketed next to the third line)
- 1 pt for substituting data from problem statement* (bracketed next to the fourth line)
- 1 pt for calculating correctly* (bracketed next to the final two lines)