Question 1 (25 points)

A multiple choice exam has 10 questions, each with 3 possible choices, and exactly one choice is the correct answer for each question. Suppose that a student guesses at random on each question.

(1) What is the probability that this student has exactly 7 correct answers?

(2) What is the probability that he has at least 7 correct answers?

Question 2 (25 points)

A truth serum has the property that 80% of the guilty suspects are properly judged. Moreover, innocent suspects are misjudged 2% of the time. If the suspect was selected from a group of suspects which only 5% have ever committed a crime, and if the serum indicates that he is guilty, what is the probability that he is innocent?

Question 3 (25 points)

Suppose we roll two die. Let $X$ be the random variable equal to the maximum number of the two die.

(1) Determine the probability mass function of $X$. Draw its graph.

(2) Compute the expectation and the variance of $X$.

Question 4 (25 points)

A byte is a string of 8 digits which can be either 0 or 1. A kbyte (kbyte) is 1024 bytes. A disk performing a certain task has a probability 0.25 of writing a 1 and 0.75 of writing a 0.

(1) What is the probability of having between 3 and 5 ones in one byte?

(2) Use the Poisson approximation to determine the probability of having exactly 1000 ones in a kbyte (write as a formula, do not calculate!).