Math 4800 – Elliptic curves (tentative syllabus)

Instructor: Gil Moss
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Schedule: Tuesday and Thursday, 2:00-3:20
Room: to be determined
Office hours: be determined

Textbook: *Rational points on elliptic curves*, by Joseph Silverman and John Tate, second edition, Springer (2015). It must be the second edition.

Description: The study of diophantine equations forms an ancient branch of number theory whose goal is to count solutions of polynomial equations in the integers or rational numbers. Fermat's last theorem is a famous example. Whereas linear and quadratic equations are well understood, much is unknown when the equations have degree 3. What little is known was proven using elegant combinations of algebra, number theory, and algebraic geometry. This course will focus in depth on cubic equations in two variables, that is, elliptic curves. We will begin by briefly covering some concepts in algebraic geometry, we will then shift to exploring the interplay of geometry and number theory in the setting of elliptic curves. In the first portion of the course, we will explore the interplay of geometry and number theory in the setting of elliptic curves. Examples of topics include finding rational points of finite order, counting points over finite fields, and the group properties of the set of rational points. The second portion of the course will shift to a broader discussion of some major developments that have taken place in the subject over the last century.

Prerequisites: enrollment is by permission of the instructor only. There are no formal prerequisites, besides having experience writing proofs. However, the course will focus on the interplay of algebra, geometry, analysis, and number theory, so it is recommended that students have some familiarity with the basic notions in these areas (e.g. linear algebra, groups, rings, fields, polynomials, infinite series, complex numbers, factorization and primes in the integers). As one example, Math 4400 would provide a good background; or taking Math 5310 concurrently.

Final project: throughout the course, we will encounter numerous topics that will be open-ended, and could be suitable for final projects. With guidance, students will choose a topic and work independently to complete a project, eventually presenting their work to the class. Part of the project will be creating a written paper typed in LaTeX.

Attendance: the theory of elliptic curves is deep and subtle, and it requires a lot of serious work to learn. Enrollment in this course constitutes a commitment to be engaged with the material and participate in the course for the entire semester. Missing one or two classes is fine, but otherwise attendance is mandatory.

Homework: there will be difficult and copious homework problems, which will be assigned and collected weekly.

Final grades: the final grade will take into account your homework, attendance, and final project. It will reflect the level of effort, participation, and engagement demonstrated by the student.

Learning objectives: by the end of the course students will

1. be able to read and understand proofs presented in an advanced undergraduate text,

2. be able to independently work through textbooks or articles to gain mastery of an open-ended topic,
3. have experience solving proof-based exercises and writing solutions clearly and correctly,
4. be able to present a topic to the class, with proofs,
5. understand the basics of elliptic curves.