Homework 8

Jeremy Morris

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Exercise 4.1 *In the three-period model of Figure 1.2.2 of Chapter 1, let the interest rate be $r = \frac{1}{4}$ so the risk-neutral probabilities are $\tilde{p} = \tilde{q} = \frac{1}{2}$.*

1. *Determine the price at time zero, denoted V_0^P, of the American put that expires at time three and has intrinsic value $g_P(s) = (4 - s)^+$.*

   The American put has price $V_0^P = 2.56$. The answer is achieved through modifying the code from project 2, code is attached.

2. *Determine the price at time zero, denoted V_0^C, of the American call that expires at time three and has intrinsic value $g_C(s) = (s - 4)^+$.*

   The American call has price $V_0^C = 0.928$.

3. *Determine the price at time zero, denoted V_0^S, of the American straddle that expires at time three and has intrinsic value $g_C(s) = g_P(s) + g_C(s)$.*

   The American straddle has price $V_0^S = 3.296$.

4. *Explain why $V_0^S < V_0^P + V_0^S$.*

   When we look at the end of the tree, we get

   $$V_3^S(\omega) = V_3^P(\omega) + V_3^S(\omega)$$

   and in the next step, we also get

   $$V_2^S(\omega) = V_2^P(\omega) + V_2^S(\omega).$$

   Then in the next to last step we have

   $$V_1^S(\omega) < V_1^P(\omega) + V_1^S(\omega).$$

   because

   $$V_1^P(T) = 2$$

   and

   $$V_1^S(T) = 2.16 < V_1^P(T) + V_1^S(T) = 2 + 0.64.$$

   What we see is that there is an added advantage to having the American put option that is not transferred to the straddle.

Exercise 4.5 *In equation (4.4.5), the maximum is computed over all stopping times in S_0. List all the stopping times in S_0 (there are 26), and from among them, list the stopping times that never exercise when the option is out of the money (there are 11). For each stopping time τ in the latter set, compute $E[I_{\{\tau \le 2\}} \frac{4}{5}^{\tau} G_\tau]$. Verify that the largest value for this quantity is given by the stopping time of (4.4.6), the one that makes this quantity equal to the 1.36 computed in (4.4.7).*

The list of all stopping times follows.

$$
\begin{array}{ll}
(0,0,0,0) & (\infty,2,\infty,\infty) \\
(1,1,1,1) & (\infty,\infty,2,\infty) \\
(2,2,2,2) & (\infty,\infty,\infty,2) \\
(\infty,\infty,\infty,\infty) & (2,2,\infty,\infty) \\
(1,1,\infty,\infty) & (2,\infty,2,\infty) \\
(1,1,2,\infty) & (2,\infty,\infty,2) \\
(1,1,\infty,2) & (\infty,2,2,\infty) \\
(1,1,2,2) & (\infty,2,\infty,2) \\
(\infty,\infty,1,1) & (\infty,\infty,2,2) \\
(2,\infty,1,1) & (2,2,2,\infty) \\
(\infty,2,1,1) & (2,2,\infty,2) \\
(2,2,1,1) & (2,\infty,2,2) \\
(2,\infty,\infty,\infty) & (\infty,2,2,2)
\end{array}
$$

The stopping times that never exercise when the option is out of the money are

$$
\begin{array}{l}
(0,0,0,0) \\
(\infty,\infty,2,\infty) \\
(\infty,\infty,\infty,2) \\
(\infty,\infty,\infty,\infty) \\
(\infty,\infty,1,1) \\
(\infty,\infty,2,2) \\
(\infty,2,\infty,\infty) \\
(\infty,2,2,\infty) \\
(\infty,2,\infty,2) \\
(\infty,2,2,2) \\
(\infty,2,1,1)
\end{array}
$$

With corresponding values for V_0 for each given by

$$
\begin{aligned}
V_0 &= 1 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1 + \left(\frac{4}{5}\right)^2 1 + 0\cdot 4\right] = \frac{1}{4}\left(\frac{4}{5}\right)^2 = 0.16 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1 + 0\cdot 1 + \left(\frac{4}{5}\right)^2 4\right] = \frac{1}{4}\left(\frac{4}{5}\right)^2 4 = 0.64 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1 + 0\cdot 1 + 0\cdot 4\right] = 0 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1\right] + \frac{1}{2}\left(\frac{4}{5}\right) 3 = 1.2 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1 + \left(\frac{4}{5}\right)^2 1 + \left(\frac{4}{5}\right)^2 4\right] = 0.8 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + 0\cdot 1 + \left(\frac{4}{5}\right)^2 1 + 0\cdot 4\right] = 0.16 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + \left(\frac{4}{5}\right)^2 1 + \left(\frac{4}{5}\right)^2 1 + 0\cdot 4\right] = 0.32 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + \left(\frac{4}{5}\right)^2 1 + 0\cdot 1 + \left(\frac{4}{5}\right)^2 4\right] = 0.8 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + \left(\frac{4}{5}\right)^2 1 + \left(\frac{4}{5}\right)^2 1 + \left(\frac{4}{5}\right)^2 4\right] = 0.96 \\
V_0 &= \frac{1}{4}\left[0\cdot 0 + \left(\frac{4}{5}\right)^2 1\right] + \frac{1}{2}\left[\left(\frac{4}{5}\right) 3\right] = 1.36
\end{aligned}
$$

Notice that we achieve the maximum when the text says we should.