

Project 2

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The purpose of this project is to demonstrate the difference in price between American and European options. The following parameters will be used for the duration of the project ($r = 0.05$, $T = 0.25$, $S_0 = 23$, $N = 50$).

1. First, the binomial model will be used to price American call and put options on a stock that pays no dividend. The difference between the American and European options will be demonstrated using a table of values representing the risk-neutral price for each. In the case of a put option, we have the following table.

K	σ	Am Price	Eu Price	Diff
21	0.10	0.00795	0.00769	0.000257
21	0.15	0.06461	0.06299	0.001614
21	0.20	0.17379	0.17029	0.003499
23	0.10	0.35347	0.32487	0.028597
23	0.15	0.57444	0.54691	0.027524
23	0.20	0.79793	0.77117	0.026758
25	0.10	2.00000	1.73023	0.269773
25	0.15	2.00909	1.85223	0.156854
25	0.20	2.12116	2.02132	0.099834

Notice that the difference between the prices is always positive, so that the American put option is always more expensive than the European put option. This demonstrates that there is an optimal time to exercise the American option before the expiration date. The next table shows values for American and European call options.

K	σ	Am Price	Eu Price	Diff
21	0.10	2.2685	2.2685	0
21	0.15	2.3238	2.3238	0
21	0.20	2.4311	2.4311	0
23	0.10	0.6106	0.6106	0
23	0.15	0.8326	0.8326	0
23	0.20	1.0569	1.0569	0
25	0.10	0.0407	0.0407	0
25	0.15	0.1628	0.1628	0
25	0.20	0.3318	0.3318	0

Here there is never a difference in price demonstrating that there is never an optimal time to exercise the option before the expiration date.

2. Next, consider a stock that pays a dividend as in exercise 2.10. Assume that $A_n = \delta \Delta t$ where $\delta = 0.06$ and Δt is the time step. The following table shows values for American and European call options.

K	σ	Am Price	Eu Price	Diff
21	0.10	2.0000	1.9352	0.064845
21	0.15	2.0492	2.0146	0.034583
21	0.20	2.1691	2.1439	0.025196
23	0.10	0.4286	0.4231	0.005480
23	0.15	0.6586	0.6526	0.005991
23	0.20	0.8872	0.8809	0.006328
25	0.10	0.0196	0.0195	0.000164
25	0.15	0.1121	0.1115	0.000635
25	0.20	0.2589	0.2577	0.001240

Now there is a difference in price, demonstrating that there is now an optimal time to exercise the American call option.