

# Homework 5

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## 1. Building an $N$ -period binomial model

The condition needed to impose on  $\Delta t$  to make it a probability measure would be that  $0 \leq \tilde{p} \leq 1$ . We solve for  $\Delta t$  in order to determine the condition.

$$\begin{aligned} 0 &\leq \frac{1}{2} \left( 1 + \left( \frac{r}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \right) \leq 1 \\ 0 &\leq 1 + \left( \frac{r}{\sigma} - \frac{\sigma}{2} \right) \sqrt{\Delta t} \leq 2 \\ -1 &\leq \left( \frac{2r - \sigma^2}{2\sigma} \right) \sqrt{\Delta t} \leq 1 \\ -\frac{2\sigma}{2r - \sigma^2} &\leq \sqrt{\Delta t} \leq \frac{2\sigma}{2r - \sigma^2} \end{aligned}$$

Of course, this does not work when  $2r = \sigma^2$ .

The code for the program is attached.

## 2. Black-Scholes and simulations

The simulations mentioned have been coded and attached here. The results are below.  
(The simulations were coded to provide output in L<sup>A</sup>T<sub>E</sub>X.)

| $K$ | $\sigma$ | $N$ | $V_N$  | Black-Scholes | Error       |
|-----|----------|-----|--------|---------------|-------------|
| 20  | 0.1      | 10  | 3.2485 | 3.2488        | -0.00032185 |
| 20  | 0.1      | 100 | 3.2487 | 3.2488        | -4.4583e-05 |
| 20  | 0.1      | 500 | 3.2488 | 3.2488        | -6.4861e-06 |
| 20  | 0.15     | 10  | 3.2579 | 3.2610        | -0.0030249  |
| 20  | 0.15     | 100 | 3.2609 | 3.2610        | -8.1121e-05 |
| 20  | 0.15     | 500 | 3.2609 | 3.2610        | -3.6200e-05 |
| 20  | 0.2      | 10  | 3.3044 | 3.3077        | -0.0032947  |
| 20  | 0.2      | 100 | 3.3067 | 3.3077        | -0.00097663 |
| 20  | 0.2      | 500 | 3.3077 | 3.3077        | 4.3604e-05  |
| 21  | 0.1      | 10  | 2.2675 | 2.2685        | -0.0010533  |
| 21  | 0.1      | 100 | 2.2684 | 2.2685        | -0.00017330 |
| 21  | 0.1      | 500 | 2.2685 | 2.2685        | -5.4881e-06 |
| 21  | 0.15     | 10  | 2.3195 | 2.3236        | -0.0041325  |
| 21  | 0.15     | 100 | 2.3230 | 2.3236        | -0.00060395 |
| 21  | 0.15     | 500 | 2.3237 | 2.3236        | 7.2629e-05  |

|    |      |     |          |          |             |
|----|------|-----|----------|----------|-------------|
| 21 | 0.2  | 10  | 2.44     | 2.431    | 0.0090116   |
| 21 | 0.2  | 100 | 2.4320   | 2.431    | 0.00098223  |
| 21 | 0.2  | 500 | 2.4309   | 2.431    | -2.8946e-05 |
| 22 | 0.1  | 10  | 1.3480   | 1.3442   | 0.0037936   |
| 22 | 0.1  | 100 | 1.3447   | 1.3442   | 0.00044204  |
| 22 | 0.1  | 500 | 1.3441   | 1.3442   | -8.5592e-05 |
| 22 | 0.15 | 10  | 1.4827   | 1.4896   | -0.006851   |
| 22 | 0.15 | 100 | 1.4886   | 1.4896   | -0.00099292 |
| 22 | 0.15 | 500 | 1.4898   | 1.4896   | 0.00019904  |
| 22 | 0.2  | 10  | 1.6825   | 1.6691   | 0.013388    |
| 22 | 0.2  | 100 | 1.6697   | 1.6691   | 0.00057887  |
| 22 | 0.2  | 500 | 1.6688   | 1.6691   | -0.00030761 |
| 23 | 0.1  | 10  | 0.60124  | 0.61291  | -0.011670   |
| 23 | 0.1  | 100 | 0.61173  | 0.61291  | -0.0011819  |
| 23 | 0.1  | 500 | 0.61267  | 0.61291  | -0.00023662 |
| 23 | 0.15 | 10  | 0.8189   | 0.83607  | -0.017167   |
| 23 | 0.15 | 100 | 0.83433  | 0.83607  | -0.0017387  |
| 23 | 0.15 | 500 | 0.83572  | 0.83607  | -0.0003481  |
| 23 | 0.2  | 10  | 1.0387   | 1.0614   | -0.022713   |
| 23 | 0.2  | 100 | 1.0591   | 1.0614   | -0.0023006  |
| 23 | 0.2  | 500 | 1.0610   | 1.0614   | -0.00046058 |
| 24 | 0.1  | 10  | 0.19949  | 0.19650  | 0.0029896   |
| 24 | 0.1  | 100 | 0.19654  | 0.19650  | 4.2369e-05  |
| 24 | 0.1  | 500 | 0.19660  | 0.19650  | 0.00010660  |
| 24 | 0.15 | 10  | 0.39609  | 0.40291  | -0.0068213  |
| 24 | 0.15 | 100 | 0.40276  | 0.40291  | -0.00014946 |
| 24 | 0.15 | 500 | 0.4031   | 0.40291  | 0.00019262  |
| 24 | 0.2  | 10  | 0.63732  | 0.62196  | 0.015359    |
| 24 | 0.2  | 100 | 0.62144  | 0.62196  | -0.0005228  |
| 24 | 0.2  | 500 | 0.62212  | 0.62196  | 0.00015936  |
| 25 | 0.1  | 10  | 0.040776 | 0.042011 | -0.0012350  |
| 25 | 0.1  | 100 | 0.041777 | 0.042011 | -0.00023406 |
| 25 | 0.1  | 500 | 0.041975 | 0.042011 | -3.6215e-05 |
| 25 | 0.15 | 10  | 0.16567  | 0.16507  | 0.00060862  |
| 25 | 0.15 | 100 | 0.16555  | 0.16507  | 0.0004814   |
| 25 | 0.15 | 500 | 0.16515  | 0.16507  | 8.8202e-05  |
| 25 | 0.2  | 10  | 0.34145  | 0.33519  | 0.0062645   |
| 25 | 0.2  | 100 | 0.33476  | 0.33519  | -0.00042605 |
| 25 | 0.2  | 500 | 0.33534  | 0.33519  | 0.00015331  |