

Homework 12

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1. *The current US treasury yield curve has an inverted shape, which implies that longer maturities have lower yields compared to short maturities. For example, the 6-month yield is 4.93%, the 2-year yield is 4.76%, and the 10-year yield is 4.59%. Let's assume these are all zero-coupon bills or bonds and continuously compounded. Find the prices of these treasury bills or bonds, assuming a facevalue \$100. Now suppose you want to price the 1-year treasury bond by interpolation using the given information about the 6-month, 2-year and 10-year treasury notes. Should you interpolate the yields, or the prices? Compute the difference and determine whether one is always above the other or the other wayaround, by studying the convexity behavior of the price-yield relationship and the shape of the yield curve.*

Using the yield equation from the text, we get

$$y_m = \left(\frac{1}{B_{0,m}} \right)^{1/m} - 1$$

so that the price of the bond is

$$B_{0,m} = \frac{1}{(1 + y_m)^m}.$$

This means that we have the following data

Maturity	Yield	Price
0.5	0.0493	97.6
2.0	0.0476	91.1
10.0	0.0459	63.8

Interpolation on the yield for the one-year bill gives $y_1 = 0.048733$, which means that the price is $B_{0,1} = 95.35$. Interpolation on the prices at the one-year maturity gives $B_{0,1} = 95.45$. The difference is 0.10 where interpolation on the prices is always larger than interpolation on the yield.

2. *The term structure of interest rates is downward-sloping (inverted). Put the following in order of magnitude:*
 - (a) *The 5-year zero rate (the yield on a zero-coupon bond);*

- (b) *The yield on a 5-year coupon-bearing bond;*
- (c) *The forward rate corresponding to the period between 5 and 5.25 years in the future.*

What is the answer to this question when the term structure of the interest rates is upward-sloping?

If the term structure is downward-sloping we have the following from lowest to highest :

- (a) The forward rate corresponding to the period between 5 and 5.25 years in the future.
- (b) The 5-year zero rate (the yield on a zero-coupon bond);
- (c) The yield on a 5-year coupon-bearing bond;

If the term structure is upward-sloping we have :

- (a) The yield on a 5-year coupon-bearing bond;
- (b) The 5-year zero rate (the yield on a zero-coupon bond);
- (c) The forward rate corresponding to the period between 5 and 5.25 years in the future.