## 8.1 Hypothesis Tests

- main goal in many research studies is to check whether the data support certain statements or predicitions.
- these statements are called hypotheses
- <u>significance test</u> : a method of using data to summarize the evidence about a hypothesis.
- <u>hypothesis</u> : statememt about a population, ususally we want to say that a parameter takes a value or falls within a range of values.
- there are 5 steps to a hypothesis test
  1. Assumptions
     - each test has certain conditions under which it is valid
     - some typical assumptions are : randomization, large sample sizes and/or specifying a distribution for the underlying data.
  2. Hypothesis
     - two hypotheses called the null ($H_0$) and the alternative ($H_a$).
     - $H_0$ typically represents no effect
  3. Test Statistic
     - usually some standardized form of the point estimate for the parameter involved in $H_0$ and $H_a$
     - we will go over a few specific cases, namely the population proportion and the population mean.
  4. P-value
     - assuming that $H_0$ is true, the p-value is the probability we would observe the test statistic or more extreme values of the test statistic
     - small p-value means that the sample data would be unusual if $H_0$ were true
  5. Conclusion
     - report the p-value and interpret it in relation to the question that motivated the test
     - make a judgement of whether or not to reject $H_0$.

## 8.2 Significance Tests for the Population Proportion

- <u>Ex 3</u> Astrology : in this study we had 116 adult volunteers, each subject had a horoscope prepared by an astrologer and filled out a CPI (California Personality Index). then an astrologer was shown birth data, horoscope and 3 CPI's (one of which corrosponded to the birth data and horoscope). then the astrologer was supposed to select the correct CPI given the information at hand. 28 participating astrologers were selected at random from a group. study was double blind, neither the astrologer or reasearcher knew the subject whose birth data and horoscope were chosen. astrologers claim that they can choose the correct personality test better than someone at random. state the null and alternative.

- in the case that we are testing the population proportion, we have the following specifics

  1. Assumptions
     - variable is categorical
     - data are obtained through randomization
     - sample size large enough for the CLT
  2. Hypothesis
     - here we have a hypothesis of the form

       $$H_0 : p = p_0 \qquad\qquad H_a : p > p_0$$

       where $p_0 = 1/3$.
     - here the alternative is used to detect whether $p$ is larger than the value specified in $H_0$
     - this is called a one sided alternative because we are only concerned with values of $p$ in $H_a$ on one side of the parameter value $p$.
     - a two-sided hypothesis would be $H_a : p \neq p_0$, here we are considering values on both sides of $p$.
  3. Test Statistic
     - calculate the $z$ score of $\hat{p}$ assuming $H_0$ is true

       $$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

     - in this example $\hat{p} = 0.34$ and $z_0 = 0.26$.
  4. P-value
     - the p-value summarizes how unusual the data would be if $H_0$ were true
     - we know that by the CLT $z_0 \sim N(0, 1)$
     - p-value is given as

       $$P\{Z > z_0\} = 1 - \Phi(z_0) = 0.3974$$

  5. Conclusion
     - summarize test by interpreting p-value
     - 0.4 not really that small
     - does not provide enough evidence against $H_0 : p = 1/3$ and in favor of $H_a : p > 1/3$.
     - the sample data are the kind that we'd expect if $p = 1/3$, or in other words an astrologer is just guessing

### How to interpret p-value

- significance test analyzes the strength of the evidence against $H_0$ (the null)
- to show that $H_a$ is preferred over $H_0$ we show that the data contradict $H_0$
- if the p-value is small, the data contradict $H_0$ in favor of $H_a$
- we never say that we have accepted $H_a$ or that we accept $H_0$, we only say that we reject $H_0$ in favor of $H_a$ or that we cannot reject $H_0$
- in the previous example we said "we cannot reject $H_0 : p = 1/3$"

### Alternative ($H_a$) determines p-value

- consider $H_0 : p = p_0 \qquad H_a : p > p_0$
- consider $H_0 : p = p_0 \qquad H_a : p < p_0$
- consider $H_0 : p = p_0 \qquad H_a : p \neq p_0$

### Ex 4 : Dogs detecting cancer

- trained dogs attempt to detect bladder cancer by smell
- the dog has 7 urine samples to choose from, one of which is positive for bladder cancer
- the dogs guessed correctly 22 of 54 times.
- test the hypothesis that the dog is better or worse than guessing.

$$H_0 : p = 1/7 \qquad\qquad H_a : p \neq 1/7$$