Math 1070
Final Exam

Name : _____

No outside materials allowed except pens or pencils and a calculator. You have all class period to finish the test. Remember to label all graphs, plots, and charts. SHOW ALL WORK.

Some things you might want to keep in mind :

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$z_{x,i} = \frac{x_i - \bar{x}}{s_x} \qquad r = \sum_{i=1}^{n} z_{x,i} z_{y,i}$$

$$a = \bar{y} - b\bar{x} \qquad b = r\left(\frac{s_y}{s_x}\right)$$

$$\hat{y} = a + bx \qquad e = y - \hat{y}$$

$$P\{X = k\} = \left(\frac{n!}{k!(n-k)!}\right) p^k (1-p)^{n-k} \qquad \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right) \qquad z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad z = \frac{x - \mu}{\sigma}$$

1. The table below classifies auto accidents by survival status (S = survived, D = died) and seat belt status of the individual involved in the accident.

	Outcome		
Belt	S	D	**Total**
Yes	412,368	510	**412,878**
No	162,527	1601	**164,128**
Total	**574,895**	**2111**	**577,006**

   (a) **(10 points)** Estimate the probability that the individual died (D) in the auto accident.

   (b) **(10 points)** Estimate the probability that the individual died, given that they
      i. wore a seat belt,
      ii. did not wear a seat belt.

   (c) **(10 points)** Are the events of dying and wearing a seat belt independent? Justify your answer mathematically.

2. When it is operating correctly, a machine for manufacturing tennis balls produces balls with a mean weight of 57.6 grams. The last eight balls manufactured had weights

$$57.3,\ 57.4,\ 57.2,\ 57.5,\ 57.4,\ 57.1,\ 57.3,\ 57.0.$$

If $s = 0.167$, use all five steps for hypothesis testing to determine whether or not the mean is 57.6. For making conclusions and/or determining critical values, let $\alpha = 0.05$.

(a) **(6 points)** Assumptions

(b) **(6 points)** Hypothesis

(c) **(6 points)** Test Statistic

(d) **(6 points)** p-value / critical value

(e) **(6 points)** Conclusion

3. We are given the variable X = the maximum of two dice. Its distribution is given as

x	**P(x)**
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

(a) **(15 points)** Show that X is a random variable.

(b) **(15 points)** Find the mean (μ) of X.

4. Use the data to answer the following questions

12 20 44 46 47 49 60 64 66 88

(a) **(20 points)** Draw a boxplot for this data.

(b) **(10 points)** Are there any outliers? Justify your answer.

5. Two candidates, Box and Draper, are running for public office. For a random sample of 400 voters in an exit poll, 230 voted for Box and 170 for Draper. Let p denote the probability that a random selected voter prefers Box. For the hypothesis

$$H_0 : p = 0.5 \qquad H_a : p > 0.5$$

do the following.

(a) **(15 points)** Calculate the test statistic z_0.

(b) **(15 points)** Make a conclusion about the outcome of the election using either the p-value or critical value if $\alpha = 0.05$.

6. Jack, Kate, Sawyer and Charlie are stranded on an island in the South Pacific. Two of them will be rescued at random.

   (a) **(15 points)** Define the sample space for this problem.

   (b) **(15 points)** What is the probability that Sawyer will be rescued?

7. The following data is found on the General Social Survey.

	Yes	No	Total
Belief in Heaven	770	388	1158
Belief in Hell	624	502	1126

Use the five steps for a hypothesis test to determine whether or not the same proportion of people believe in heaven or hell. (Let $\alpha = 0.05$.)

(a) **(6 points)** Assumptions

(b) **(6 points)** Hypothesis

(c) **(6 points)** Test Statistic

(d) **(6 points)** p-value / critical value

(e) **(6 points)** Conclusion

8. Owen would like to get a Ph.D. in Mathematics from Cornell. He needs to take the GRE Math Subject Test as part of the application process. The Math Subject test is normally distributed with $\mu = 615$ and $\sigma = 20$.

   (a) **(15 points)** Owen needs to score in the 30^{th} percentile to be accepted. What score does he need?

   (b) **(15 points)** What is the probability that Owen scores above 700?

9. In the 2000 General Social Survey, respondents were asked whether they favored or opposed the death penalty for people convicted of murder. There were 2565 respondents and 1764 of them answered that they were in favor of the death penalty.

   (a) **(15 points)** Find a 95% confidence interval for the proportion (p) of those who are in favor of the death penalty.

   (b) **(15 points)** Interpret this confidence interval in context.

10. The table below shows summary data for sales of the Palm M515 PDA for a one week selling period in May 2003.

Variable	**Buy-it-now**	**n**	**Mean**	**St Dev**
Price	No	136	223.36	23.74
	Yes	132	235.45	20.57

If $\alpha = 0.05$, do the following.

(a) **(10 points)** Calculate the test statistic t_0 for the hypothesis

$$H_0 : \mu_1 = \mu_2 \qquad H_a : \mu_1 \neq \mu_2$$

where μ_1 is the population mean for Buy-it-now purchases and μ_2 is the population mean for regular auctions.

(b) **(10 points)** Calculate the p-value or critical value.

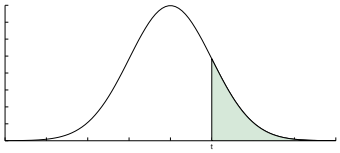
(c) **(10 points)** Make a conclusion and interpret the result.

Cummulative Probabilities for the Standard Normal Distribution

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
-0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414

Cummulative Probabilities for the t Distribution



Confidence Level	0.80	0.90	0.95	0.98	0.99	0.998
df	$t_{0.1}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	$t_{0.001}$
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
50	1.299	1.676	2.009	2.403	2.678	3.261
60	1.296	1.671	2.000	2.390	2.660	3.232
80	1.292	1.664	1.990	2.374	2.639	3.195
100	1.290	1.660	1.984	2.364	2.626	3.174
∞	1.282	1.645	1.960	2.326	2.576	3.091