5.3 Conditional Probabilities

1. we have new notation for the conditional proportions (or probabilities) that we talked about in ch 3

$$P\{A|B\} = \frac{P\{A \text{ and } B\}}{P\{B\}}$$

2. we will use the new ideas from probability that we have learned to determine whether two events are independent or not.

3. example : school children rehash. we have the table of probabilities for $P\{A \text{ and } B\}$ where $A = \{\text{educational goals}\}$ and $B = \{\text{gender}\}$.

	Grades	Popular	Sports	Total
boy	0.245	0.105	0.125	0.475
girl	0.272	0.190	0.062	0.525
Total	0.517	0.295	0.188	1.000

then we have the table of conditional probabilities $P\{A|B\}$.

	Grades	Popular	Sports
boy	0.515	0.220	0.264
girl	0.518	0.363	0.120

4. we can also use conditional probabilities to determine whether two events are independent. we have

$$\begin{aligned} P\{A|B\} &= \frac{P\{A \text{ and } B\}}{P\{B\}} \\ &= \frac{P\{A\}P\{B\}}{P\{B\}} \qquad \text{(independence)} \\ P\{A|B\} &= P\{A\} \end{aligned}$$

5. example : return to school children
   (a) let $A = \{\text{goal} = \text{Grades}\}$ and $B = \{\text{gender} = \text{Boy}\}$. are A and B independent?
   (b) let $A = \{\text{goal} = \text{Popular}\}$ and $B = \{\text{gender} = \text{Boy}\}$. are A and B independent?
   (c) let $A = \{\text{goal} = \text{Sports}\}$ and $B = \{\text{gender} = \text{Boy}\}$. are A and B independent?

6. there are three ways to check for independence
   (a) is $P\{A|B\} = P\{A\}$?
   (b) is $P\{B|A\} = P\{B\}$?
   (c) is $P\{A \text{ and } B\} = P\{A\}P\{B\}$?

   if the answer is yes to any of them, then the events A and B are independent.

7. multiplication rule : we can use the definition of conditional probabilities to calculate the probability of events A and B.

$$P\{A \text{ and } B\} = P\{A|B\}P\{B\}$$

8. example Roger Federer, Wimbeldon 2004 made 64% of his first serves (meaning he missed 36% of them). knowing that he faulted on the first, he faulted 6% of the time on his second attempt. find the probability of a double fault.

9. sampling without replacement : choosing subjects from a population where subjects are removed from the population after being selected.

10. example : Lotto south chooses 6 number 1 - 49, sampled without replacement. find the probability of winning.

5.4 More Examples

1. ex : space shuttle safety

   out of 113 missions, there were 2 failures. find $P\{\text{at least one failure in 100 missions}\}$.

   let A = at least one failure in 100 missions. then A^c = no failures in 100 missions and $P\{A\} = 1 - P\{A^c\}$. then let $S1$ = successful 1st mission, $S2$ = sucessful 2nd mission and so on. if $P\{S\} = 0.971$ then $P\{A\} = 0.947$ and if $P\{S\} = 0.9999833$ then $P\{A\} = 0.002$. the probability depends a lot on the assumed probability of success.

2. diagnostic testing : let S = some state actually present (i.e. taking drugs, some medical condition actually present), POS = tested positive for the condition, NEG = tested negative for the condition. then we have the following conditional probabilities

	Test Result	
State Present	Positive (POS)	Negative (NEG)
Yes (S)	sensitivity $P\{POS|S\}$	false negative $P\{NEG|S\}$
No (S^c)	false positive $P\{POS|S^c\}$	specificity $P\{NEG|S^c\}$

3. given the state is present, sensitivity is the probability the test detects it (by giving a positive result)

4. given the state is not present, specificity is the probability the test gives a negative result

5. example : drug testing in air traffic controllers. we are given the following for a drug test given to air traffic controllers : specificity = 0.93, sensitivity = 0.96 and prevalence (actual drug use) = 0.007.

   (a) find the probability of a positive test

(b) find the probability an air traffic controller used drugs given they test positive.

(c) find the probability an air traffic controller that tested positive was actually using drugs.