5.1 Probability

1. probability : quantifying uncertainty

2. random phenomena : many things for which the outcome is uncertain

3. ex : boardgame after 100 rolls of the die a 6 appears 23 times, at one time there are 3 6's in a row. your opponent complains that the die is loaded.

   (a) *if a fair die is rolled 100 times, how many 6's do you expect?*

   the probability of rolling a 6 on a fair die is 1/6, so that we expect $100 \times 1/6 = 16.7 \approx 17$ 6's to be rolled.

   (b) *would it be unusual to get 23 6's out of 100 trials? would it be surprising to roll 3 6's in a row?*

   the text suggests using simulations to answer this question. we could run a simulation where we roll a die 100 times and keep track (after each roll or trial) of how many 6's have appeared up to that point. the authors performed this simulation and determined that in 100 trials it is not impossible for 23 6's to be rolled in 100 trials or to get a run of 3 6's. this one simulation does not prove anything concrete, but shows that the outcome is not impossible.

4. why do we see this behavior? probabilities tell us what the long run behavior of the random phenomena should be. short run behavior can be very unpredictable.

5. for example, if we were to perform the simulation only 10 times, we could see very strange behavior. however, if we were to do the simulation 10,000 times, the cummulative proportion of 6's would approach 1/6.

6. ex : weather when a forcaster says that there is a 70% chance of rain, what he/she means is that on a large number of days w/ the same atmospheric conditions, the proportion of days that it rained was 70%.

7. independent trials : when the outcome of any one trial does not depend on the outcome of any other.

8. ex : dice if we roll a die once and get a 6, what is the chance of getting a 6 the second time? is it different because we got a 6 the first time?

9. ex : monopoly

10. how can we find probabilities : make assumptions about the phenomena we are studying. (e.g. assume that each side of the die is equally likely).

11. types of probability

    (a) objective probabilities : the kind we have been talking about, can be explicitly calculated after making some assumptions about the data.

(b) subjective probabilities : defined by your degree of belief about the outcome of some random event. (e.g. the grade you will earn in this class counts as subjective.) there is a branch of probability theory called baysian statistics that deals with subjective probabilities.

5.2 Finding Probabilities

1. sample space : the set of all possible outcomes.
2. example
   (a) roll a die once : $\{1, 2, 3, 4, 5, 6\}$
   (b) flip a coin twice : $\{HH, HT, TH, TT\}$
      - we could use a tree diagram to list the outcomes in the sample space. we alter 2b to flip a coin three times and write out the tree.
      - here the point is to calculate the number of outcomes in the sample space.

   (c) flip a coin twice and roll a die once. what is the number of outcomes. $*, *, * =>$ $2 \cdot 2 \cdot 6 = 24$ outcomes.
   (d) multiple choice exam. 10 questions, 6 w/ 5 options, 2 w/ 3 options and 2 w/ 2 options. then we get $5^6 3^2 2^2 = 562500$ outcomes in the sample space. if we were to list all of the ways that this exam could be answered, we would have to list that many.
3. event : a subset of the sample space. usually denoted with a capital letter. in this case we say that an event A is a subset of the sample space S.
4. examples :
   (a) $A = \{\text{die lands on an even number}\}$: $\{2, 4, 6\}$.
   (b) $A = \{\text{exactly two heads in three flips}\}$: $\{HHT, HTH, THH\}$.
5. equation for finding probabilities

$$P\{A\} = \frac{\#\text{ of outcomes in event } A}{\#\text{ of outcomes in the sample space}}$$

   (a) probabilities are between 0 and 1
6. example :
   (a) $A =$ die lands even. Find $P\{A\}$.
   (b) $A =$ exactly two heads in three flips of the coin. Find $P\{A\}$.
   (c) $A =$ exactly one head in two flips of the coin. Find $P\{A\}$.
7. Ex 4 UW is going to test an herbal remedy against a placebo. they have 4 volunteers Jamal, Ken, Linda and Mary. two will be randomly chosen to recieve the herbal remedy and two the placebo.

(a) find the sample space to recieve the placebo.

(b) what is the probability that the placebo group has one woman and one man? (either Jamal or Ken, but not both)