

Homework 2

Let X_t satisfy

$$\varphi(B)X_t = \theta(B)Z_t, \quad -\infty < t < \infty$$

where $\{Z_t\}$ is White Noise $(0, \sigma^2)$. Find the representation

$$X_t = \sum_{i=0}^{\infty} \psi_i Z_{t-i} \quad (\text{casual})$$

and

$$Z_t = \sum_{j=0}^{\infty} \pi_j X_{t-j} \quad (\text{invertable})$$

in the following cases:

1.

$$\varphi(z) = 1 - z + \frac{1}{4}z^2 \quad \text{and} \quad \theta(z) = 1.$$

2.

$$\varphi(z) = 1 + \frac{1}{9}z^2 \quad \text{and} \quad \theta(z) = 1 + \frac{1}{3}z.$$

3.

$$\varphi(z) = (1 - \frac{1}{3}z)(1 - \frac{1}{2}z)(1 - \frac{1}{4}z) \quad \text{and} \quad \theta(z) = 1.$$

4.

$$\varphi(z) = (1 + \frac{1}{2}z)(1 - \frac{1}{5}z) \quad \text{and} \quad \theta(z) = (1 - \frac{1}{2}z)(1 - \frac{1}{3}z)$$

5.

$$\varphi(z) = (1 + \frac{1}{2}z)^3 \quad \text{and} \quad \theta(z) = 1.$$