Ch 6 Homework

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April 17, 2006

6.5 *For fixed factors, explain the trade-offs between selection of levels and the expected detectability.*

Levels should be chosen so that the intervals given by the expected detectability do not overlap. This will vary depending on the experiment. Levels should also not be chosen so far apart that they lose any meaning.

6.6 *Explain how many levels should be selected for factors that are random, factors that are fixed and quantitative and factors that are fixed and qualitative.*

Any number of random factors can be chosen, the idea would be to balance the size of the experiment and the desired detectability. The number of levels for fixed quantitative factors should be chosen so that any assumed relationship would be captured. In other words, if there is a possibility of a quadratic relationship, at least three levels should be chosen. Qualitative, fixed factors should be chosen according to how many categories exist.

6.7 *Suppose two different suppliers supplied Injectors for this engine. Explain how you would change the experiment to include the suppliers in the inference space.*

Add a factor called Supplier to the model and nest Injectors with Supplier.

6.10 *The trick of putting the restriction on the EF interaction worked very well in this experiment because tests on E and F still existed. Would this trick work if one of these factors was fixed? Would it work if both are fixed?*

The following table shows the tests for the original model where E and F were random.

Source	df	EMS	Tested With
E_i	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2 + 2\sigma_{EI}^2 + 8\sigma_E^2$	$EI_{ij} + EF_{ik} - EIF_{ijk}$
I_j	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_{IF}^2 + 2\sigma_{EI}^2 + 8\sigma_I^2$	$EI_{ij} + IF_{jk} - EIF_{ijk}$
EI_{ij}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 2\sigma_{EI}^2$	EIF_{ijk}
F_k	1	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_{IF}^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2 + 16\sigma_F^2$	$EF_{ik} + IF_{jk} - EIF_{ijk}$
EF_{ik}	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2$	$--$
$\delta_{m(ik)}$	0	$--$	$--$
IF_{jk}	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_{IF}^2$	EIF_{ijk}
EIF_{ijk}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2$	$--$
$\varepsilon_{l(ijk)}$	0	σ_ε^2	$--$

This changes to the following if E_i is fixed.

Source	df	EMS	Tested With
E_i	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2 + 2\sigma_{EI}^2 + 8\Phi(E)$	$EI_{ij} + EF_{ik} - EIF_{ijk}$
I_j	3	$\sigma_\varepsilon^2 + 4\sigma_{IF}^2 + 8\sigma_I^2$	IF_{ik}
EI_{ij}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 2\sigma_{EI}^2$	EIF_{ijk}
F_k	1	$\sigma_\varepsilon^2 + 4\sigma_{IF}^2 + 4\sigma_\delta^2 + 16\sigma_F^2$	$--$
EF_{ik}	3	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2$	$--$
$\delta_{m(ik)}$	0	$--$	$--$
IF_{jk}	3	$\sigma_\varepsilon^2 + 4\sigma_{IF}^2$	$--$
EIF_{ijk}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2$	$--$
$\varepsilon_{l(ijk)}$	0	σ_ε^2	$--$

Notice that there is now a direct test for I_j, there is still an approximate test for E_i, there is only a conservative test for F_k and there is no longer a test for IF_{ik}. The following table contains the factors and tests if both E_i and I_j are fixed.

Source	df	EMS	Tested With
E_i	3	$\sigma_\varepsilon^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2 + 8\Phi(E)$	EF_{ik}
I_j	3	$\sigma_\varepsilon^2 + 4\sigma_{IF}^2 + 8\Phi(I)$	IF_{jk}
EI_{ij}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2 + 2\Phi(EI)$	EIF_{ijk}
F_k	1	$\sigma_\varepsilon^2 + 4\sigma_\delta^2 + 16\sigma_F^2$	$--$
EF_{ik}	3	$\sigma_\varepsilon^2 + 4\sigma_\delta^2 + 4\sigma_{EF}^2$	$--$
$\delta_{m(ik)}$	0	$--$	$--$
IF_{jk}	3	$\sigma_\varepsilon^2 + 4\sigma_{IF}^2$	$--$
EIF_{ijk}	9	$\sigma_\varepsilon^2 + \sigma_{EIF}^2$	$--$
$\varepsilon_{l(ijk)}$	0	σ_ε^2	$--$

Now we have direct tests for E_i, I_j and EI_{ij}, conservative tests for F_k and EF_{ik} and no tests for IF_{jk} or EIF_{ijk}.

6.15 *Figure 6.4 plots the raw data for each Injector. These plots are comonly used, but not recommended. Why not?*

A plot of the raw data as in (6.4) does not take into account variance due to other effects, whereas the corresponding plot in (6.3) which plots the means along with a confidence interval gives a much more realistic picture of what is going on with the mean.

6.22 *To try and get the symmetry back into the Engine/Injector/Fuel example, consider running Design 4 with two blocks of two different fuels each. Compare the power of this design to the ANOVA in table 6.10.2.*

Here is the table describing how the tests will be performed for the layout including blocking.

Source	df	EMS	F_{calc} denominator
E_i	3	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{EF}^2+2\sigma_{EIB}^2+8\sigma_{EB}^2+4\sigma_{EI}^2+16\sigma_E^2$	$EI_{ij}+EB_{ik}-EIB_{ijk}$
I_j	3	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{IF}^2+2\sigma_{EIB}^2+8\sigma_{IB}^2+4\sigma_{EI}^2+16\sigma_I^2$	$EI_{ij}+IB_{jk}-EIB_{ijk}$
EI_{ij}	9	$\sigma_\varepsilon^2+\sigma_{EIF}^2+2\sigma_{EIB}^2+4\sigma_{EI}^2$	EIB_{ijk}
B_k	1	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{IF}^2+4\sigma_{EF}^2+16\sigma_F^2+2\sigma_{EIB}^2+$ $8\sigma_{IB}^2+8\sigma_{EB}^2+32\sigma_\delta^2+32\sigma_B^2$	- -
$\delta_{o(k)}$	0	$--$	- -
EB_{ik}	3	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{EF}^2+2\sigma_{EIB}^2+8\sigma_{EB}^2$	$EIB_{ijk}+EF_{im(k)}-EIF_{ijm(k)}$
IB_{jk}	3	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{IF}^2+2\sigma_{EIB}^2+8\sigma_{IB}^2$	$EIB_{ijk}+IF_{jm(k)}-EIF_{ijm(k)}$
EIB_{ijk}	9	$\sigma_\varepsilon^2+\sigma_{EIF}^2+2\sigma_{EIB}^2$	$EIF_{ijm(k)}$
$F_{m(k)}$	2	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{IF}^2+4\sigma_{EF}^2+16\sigma_F^2$	$EF_{im(k)}+IF_{jm(k)}-EIF_{ijm(k)}$
$EF_{im(k)}$	6	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{EF}^2$	$EIF_{ijm(k)}$
$IF_{jm(k)}$	6	$\sigma_\varepsilon^2+\sigma_{EIF}^2+4\sigma_{IF}^2$	$EIF_{ijm(k)}$
$EIF_{ijm(k)}$	8	$\sigma_\varepsilon^2+\sigma_{EIF}^2$	- -
$\varepsilon_{n(ijkm)}$	0	$--$	- -

This table compares Design 4 with the design proposed in this problem. The table shows that by blocking on the fuel we can decrease the size of the minimum detectibility. However, this doubles the number of experiments.

	Design 4			Problem 6.22		
Source	df	Δ	Size	df	Δ	Size
E_i	3	1.86	medium	3	1.32	small
I_j	3	1.86	medium	3	1.32	small
EI_{ij}	9	1.84	medium	9	1.30	small
B_k				1	$--$	$--$
$\delta_{o(k)}$				0	$--$	$--$
EB_{ik}				3	1.60	medium
IB_{jk}				3	1.60	medium
EIB_{ijk}				9	1.53	medium
$F_{m(k)}$	1	5.82	ext. large	2	1.64	medium
$EF_{im(k)}$	3	2.19	medium	6	1.28	small
$IF_{jm(k)}$	3	2.19	medium	6	1.28	small
$EIF_{ijm(k)}$	9	$--$	$--$	8	$--$	$--$
$\varepsilon_{n(ijkm)}$	0	$--$	$--$	0	$--$	$--$