Chapter 5 Homework

Jeremy Morris

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5.1 *Reconsider the cardiac valve experiment of Chapter 4 with four types of valves, two valves of each type, six different pulse rates, and two repeats of each combination. Identify the restriction errors for each of the following layouts.*

(a) *A pulse rate is fixed, the eight valves (two each of four different types) are randomly ordered, and the repeat measurements are taken back to back.*

There is a restriction on the pulse rate and one on the combination of pulse rate and valve.

(b) *A pulse rate is fixed and the sixteen measurements (two each for the two different valves of each type) are randomly ordered and run.*

There is a restriction on the pulse rate.

(c) *One of the eight valves is randomly selected and the twelve runs (two repeats of each of the six pulse rates) are randomly ordered and run.*

There is a restriction on the valve and valve/pulse rate interaction.

(d) *A valve type and one of the two valves is randomly selected. The twelve runs (two repeats of the six pulse rates) are randomly run before the other valve of the same type is selected.*

This is a restriction on the valve.

(e) *One of the eight valves and a pulse rate is selected and measured twice. The other pulse rates are tested before changing the valve.*

There is a restriction on the valve, pulse rate interaction.

(f) *A valve and a pulse rate is randomly selected with the repeat measurements back to back.*

This is a restriction on the valve/pulse rate interaction.

(g) *The 96 combinations are written down on a piece of paper with the repeats adjacent to each other and the pulse rates ordered on through six. These are labeled 1 to 96. A random number table is used to determine which label is run first, which label is run second and so on.*

No restrictions.

(h) *One of the twelve type/pulse rates combinations is randomly selected. One of the two valves is then selected and both repeats run before testing the other valve.*

Restriction on the interaction between valve type and pulse rate. There is also a restriction on the valve/pulse rate interaction.

5.3 *Describe how the experiment should be run for each of the following math models.*

(a) $y_{ijkl} = \mu + A_i + B_j + \delta_{k(j)} + AB_{ij} + \varepsilon_{l(ijk)}$
Randomly select a level of B, randomly run repeats of A before changing levels of B.

(b) $y_{ijkl} = \mu + A_i + \delta_{j(i)} + B_k + AB_{ik} + \varepsilon_{l(ijk)}$
Select a level of A, randomly run repeats of B before changing levels of A.

(c) $y_{ijkl} = \mu + A_i + B_j + AB_{ij} + \delta_{k(ij)} + \varepsilon_{l(ijk)}$
Select the combination of one level of A and one level of B, do reps before selecting the next combination of A and B.

(d) $y_{ijklm} = \mu + A_i + \delta_{j(i)} + B_k + AB_{ik} + \gamma_{l(ik)} + \varepsilon_{m(ijkl)}$
Select a level of A, run reps of B before moving on to the next level of B. Change level of A and repeat.

(e) $y_{ijklmno} = \mu + A_i + B_{j(i)} + \delta_{k(ij)} + C_{l(ij)} + D_{m(ijl)} + \gamma_{n(ijlm)} + \varepsilon_{o(ijklm)}$
The nesting here implies that for each level of A, there are different levels of B. For each level of B there are different levels of C and for each level of C there are different levels of D. That being said, the experiments will be run by selecting the A/B combination, then running the reps one after the other before switching to a different C/D combination.

5.8 *Suppose an experiment has factors called subjects and time after exposure. Explain how the experiment could be completely randomized. Why is this not generally a good idea?*

Each subject would have to perform the experiment for each time after exposure. This may not be the ideal situation because each exposure could affect the subsequent measurements that will be taken.

5.17 *Compute the ANOVA table for a completely randomized experiment having I treatments and J repeats and compare it to Table 5.4.1. What does this say about the process known as blocking? This is an important concept to learn.*

The ANOVA table for the completely randomized experiment follows.

Source	df	EMS
T_i	$I-1$	$\sigma_\varepsilon^2 + J\Phi(T)$
$\varepsilon_{j(i)}$	$I(J-1)$	σ_ε^2

It will be compared to table *5.4.1* which I have copied here.

Source	df	EMS
T_i	$I-1$	$\sigma^2 + \sigma_{TB}^2 + J\Phi(T)$
B_j	$J-1$	$\sigma^2 + I\sigma_\delta^2 + I\sigma_B^2$
$\delta_{k(j)}$	0	$--$
TB_{ij}	$(I-1)(J-1)$	$\sigma^2 + \sigma_{TB}^2$
$\varepsilon_{l(ijk)}$	0	$--$

The One-Way table appears to be better since the error term σ_ε^2 has more degrees of freedom than the interaction term TB_{ij}, which tests the treatment in the Blocked experiment. However, if the blocking exists σ_ε^2 includes the error terms for TB_{ij} and B_j and is, therefore, larger than the error term in the blocked model. Before accepting each design, the detectable difference should be calculated to make a sound judgement between the models.

5.18 *Use the rules in Chapter 3 to find the standard error for a single treatment mean in Example 1. Also find the standard error for the difference of two means and for multiple comparisons. This is a somewhat surprising result that seems counterintuitive.*

$$\sigma_{\bar{y}_T} = \sqrt{\frac{\sigma^2}{JKL} + \frac{\sigma_B^2}{J} + \frac{\sigma_\delta^2}{IJ} + \frac{(I-1)\sigma_{TB}^2}{IJ}} \quad (1)$$

$$\sigma_{\bar{y}_{T1}-\bar{y}_{T2}} = \sqrt{\frac{\sigma^2 + \sigma_{TB}^2}{I}} \quad (2)$$

This result is counterintuitive because the standard error for a single mean ($\sigma_{\bar{y}_T}$) cannot be estimated without knowing something about the error term, where as the other can be since $\sigma^2 + \sigma_{TB}^2 \approx MS(TB)$.

5.21 *An experiment considered the effect of the hydrocarbons and barometric pressure on the weight gain of laboratory rats. Three levels of hydrocarbon were used in conjunction with two levels of barometric pressure. Six controlled chambers, one for each hydrocarbon/barometric pressure combination, were used. Five rats were placed in each chamber and the change in weight measured for five consecutive weeks. Write down the model describing this experiment and indicate all tests.*

The model is summarized below with tests indicated by arrows. We need to recognize Weeks as a factor and the fact that the rats are nested within the hydrocarbon/barometric pressure combination.

Source	df	EMS
H_i	2	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 50\sigma_{HC}^2 + 50\sigma_{HP}^2 + 100\sigma_H^2$
P_j	1	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 75\sigma_{PC}^2 + 50\sigma_{HP}^2 + 150\sigma_P^2$
HP_{ij}	2	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 50\sigma_{HP}^2$
C_k	1	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 75\sigma_{PC}^2 + 50\sigma_{HC}^2 + 150\sigma_C^2$
HC_{ik}	2	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 50\sigma_{HC}^2$
PC_{jk}	1	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2 + 75\sigma_{PC}^2$
HPC_{ijk}	2	$\sigma_\varepsilon^2 + 5\sigma_R^2 + 25\sigma_{HPC}^2$
$R_{l(ijk)}$	48	$\sigma_\varepsilon^2 + 5\sigma_R^2$
W_m	4	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 15\sigma_{PCW}^2 + 10\sigma_{HCW}^2 + 30\sigma_{CW}^2 + 10\sigma_{HPW}^2 + 30\sigma_{PW}^2 + 20\sigma_{HW}^2 + 60\sigma_\delta^2 + 60\sigma_W^2$
$\delta_{n(m)}$	0	$\sigma_\varepsilon^2 + 60\sigma_\delta^2$
HW_{im}	8	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 10\sigma_{HCW}^2 + 10\sigma_{HPW}^2 + 20\sigma_{HW}^2$
PW_{jm}	4	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 15\sigma_{PCW}^2 + 10\sigma_{HPW}^2 + 30\sigma_{PW}^2$
HPW_{ijm}	8	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 10\sigma_{HPW}^2$
CW_{km}	4	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 15\sigma_{PCW}^2 + 10\sigma_{HCW}^2 + 30\sigma_{CW}^2$
HCW_{ikm}	8	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 10\sigma_{HCW}^2$
PCW_{jkm}	4	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2 + 15\sigma_{PCW}^2$
$HPCW_{ijkm}$	8	$\sigma_\varepsilon^2 + \sigma_{RW}^2 + 5\sigma_{HPCW}^2$
$RW_{lm(ijk)}$	92	$\sigma_\varepsilon^2 + \sigma_{RW}^2$
$\varepsilon_{o(ijklm)}$	0	σ_ε^2

5.36 *In many textbooks, authors talk about "repeating and experiment" and "replicating an experiment." Physically, "repeating an experiment" means that a factor combination is set up and all repeats are run back to back. "Replicating an experiment" refers to running all combinations again, generally at a later time period. Indicate where the restriction error occurs in both models and give a meaning for each restriction error.*

"Repeating an experiment" means that there will be a restriction error on the highest order interaction. "Replicating an experiment" means that there will be an extra factor in the model that represents time and there will be a restriction error related to this factor.

The restriction error related to "repeating an experiment" will be included in the EMS for every other factor and interaction and represents the error involved in breaking down and setting up the experiments.