Ch 4 Homework

Jeremy Morris

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4.2 *An experiment consists of fuel injectors, I, engines, E, and atmospheric conditions represented by weeks, W. One fuel injector is put in one engine and the vehicle is tested. Assuming complete randomization, identify the nesting in each of the following experimental layouts.* (4.19) *Assuming engines, injectors and weeks are all random, determine the tests for each of the experimental layouts given in Problem 4.2.*

(a) *Four engines and twelve injectors are selected. Three different injectors are tested in each engine. A week later, the exact procedure is repeated with the same engines and injectors.*

Injectors are nested in engines. The model becomes :

$$y_{ijkl} = \mu + W_i + E_j + WE_{ij} + I_{k(j)} + WI_{ik(j)} + \varepsilon_{(ijk)} \quad (1)$$

Source	EMS	Tested By
W_i	$\sigma_\varepsilon^2 + \sigma_{WI}^2 + K\sigma_{WE}^2 + JK\sigma_W^2$	$MS(WE)$
E_j	$\sigma_\varepsilon^2 + \sigma_{WI}^2 + I\sigma_I^2 + K\sigma_{WE}^2 + IK\sigma_E^2$	$MS(I) + MS(WE) - MS(WI)$
WE_{ij}	$\sigma_\varepsilon^2 + \sigma_{WI}^2 + K\sigma_{WE}^2$	$MS(WI)$
$I_{k(j)}$	$\sigma_\varepsilon^2 + \sigma_{WI}^2 + I\sigma_I^2$	$MS(WI)$
$WI_{ik(j)}$	$\sigma_\varepsilon^2 + \sigma_{WI}^2$	$--$
$\varepsilon_{(ijk)}$	σ_ε^2	$--$

(b) *Four engines and eight injectors are selected. Four injectors are selected and tested in all four engines. A week later, the four remaining injectors are tested in the same four engines.*

Injectors are nested in weeks. Then we have the model :

$$y_{ijkl} = \mu + W_i + E_j + WE_{ij} + I_{k(i)} + EI_{jk(i)} + \varepsilon_{(ijk)} \quad (2)$$

Source	EMS	Tested By
W_i	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + J\sigma_I^2 + IK\sigma_{WE}^2 + JK\sigma_W^2$	$MS(WE) + MS(I) - MS(EI)$
E_j	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + K\sigma_{WE}^2 + IK\sigma_E^2$	$MS(WE)$
WE_{ij}	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + K\sigma_{WE}^2$	$MS(EI)$
$I_{k(i)}$	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + J\sigma_I^2$	$MS(EI)$
$EI_{jk(i)}$	$\sigma_\varepsilon^2 + \sigma_{EI}^2$	$--$
$\varepsilon_{(ijk)}$	σ_ε^2	$--$

(c) *Four engines and eight injectors are selected. Two engines and four injectors are selected, each engine being tested with two different injectors. A week later, the experiment is repeated with two new engines and four new injectors.*

Engines and injectors are both nested in weeks and injectors are nested within engines, leaving us with the following model.

$$y_{ijkl} = \mu + W_i + E_{j(i)} + I_{k(ij)} + \varepsilon_{(ijk)} \tag{3}$$

Source	EMS	Tested By
W_i	$\sigma^2_\varepsilon + \sigma^2_I + K\sigma^2_E + JK\sigma^2_W$	$MS(E)$
$E_{j(i)}$	$\sigma^2_\varepsilon + \sigma^2_I + K\sigma^2_E$	$MS(I)$
$I_{k(ij)}$	$\sigma^2_\varepsilon + \sigma^2_I$	$--$
$\varepsilon_{(ijk)}$	σ^2_ε	$--$

(d) *Four engines and four injectors are selected. All four injectors are tested in two of the engines the first week. The second week, the same injectors are put in each of the other two engines.*

Engines are nested within weeks. The model follows.

$$y_{ijkl} = \mu + W_i + E_{j(i)} + I_k + WI_{ik} + EI_{jk(i)} + \varepsilon_{(ijk)} \tag{4}$$

Source	EMS	Tested By
W_i	$\sigma^2_\varepsilon + \sigma^2_{EI} + J\sigma^2_{WI} + K\sigma^2_E + JK\sigma^2_W$	$MS(E) + MS(WI) - MS(EI)$
$E_{j(i)}$	$\sigma^2_\varepsilon + \sigma^2_{EI} + K\sigma^2_E$	$MS(EI)$
I_k	$\sigma^2_\varepsilon + \sigma^2_{EI} + J\sigma^2_{WI} + IJ\sigma^2_I$	$MS(WI)$
WI_{ik}	$\sigma^2_\varepsilon + \sigma^2_{EI} + J\sigma^2_{WI}$	$MS(EI)$
$EI_{jk(i)}$	$\sigma^2_\varepsilon + \sigma^2_{EI}$	$--$
$\varepsilon_{(ijk)}$	σ^2_ε	$--$

(e) *Four engines and four injectors are selected. All four injectors are tested in each engine. A week later, all four injectors are tested in each engine again.*

No nesting.

$$y_{ijkl} = \mu + W_i + E_j + WE_{ij} + I_k + WI_{ik} + EI_{jk} + WEI_{ijk} + \varepsilon_{(ijk)} \tag{5}$$

Source	EMS	Tested By
W_i	$\sigma^2_\varepsilon + \sigma^2_{WEI} + J\sigma^2_{WI} + K\sigma^2_{WE} + JK\sigma^2_W$	$MS(WE) + MS(WI) - MS(WEI)$
E_j	$\sigma^2_\varepsilon + \sigma^2_{WEI} + I\sigma^2_{EI} + K\sigma^2_{WE} + IK\sigma^2_E$	$MS(WE) + MS(EI) - MS(WEI)$
WE_{ij}	$\sigma^2_\varepsilon + \sigma^2_{WEI} + K\sigma^2_{WE}$	$MS(WEI)$
I_k	$\sigma^2_\varepsilon + \sigma^2_{WEI} + I\sigma^2_{EI} + J\sigma^2_{WI} + IJ\sigma^2_I$	$MS(WI) + MS(EI) - MS(WEI)$
WI_{ik}	$\sigma^2_\varepsilon + \sigma^2_{WEI} + J\sigma^2_{WI}$	$MS(WEI)$
EI_{jk}	$\sigma^2_\varepsilon + \sigma^2_{WEI} + I\sigma^2_{EI}$	$MS(WEI)$
WEI_{ijk}	$\sigma^2_\varepsilon + \sigma^2_{WEI}$	$--$
$\varepsilon_{(ijk)}$	σ^2_ε	$--$

(f) *Two engines and eight injectors are selected. Four injectors are chosen, two to be tested in each engine. A week later, the other four injectors are tested in the same engines, again two injectors per engine.*

Injectors are nested within weeks and engines.

$$y_{ijkl} = \mu + W_i + E_j + WE_{ij} + I_{k(ij)} + \varepsilon_{(ijk)} \tag{6}$$

Source	EMS	Tested By
W_i	$\sigma_\varepsilon^2 + \sigma_I^2 + K\sigma_{WE}^2 + JK\sigma_W^2$	$MS(WE)$
E_j	$\sigma_\varepsilon^2 + \sigma_I^2 + K\sigma_{WE}^2 + IK\sigma_E^2$	$MS(WE)$
WE_{ij}	$\sigma_\varepsilon^2 + \sigma_I^2 + K\sigma_{WE}^2$	$MS(I)$
$I_{k(ij)}$	$\sigma_\varepsilon^2 + \sigma_I^2$	−−
$\varepsilon_{(ijk)}$	σ_ε^2	−−

(g) *Four engines and four injectors are chosen. Two injectors are used in each of two engines the first week. The next week, the other two injectors are used each of the other two engines.*

Engines and injectors are nested within weeks.

$$y_{ijkl} = \mu + W_i + E_{j(i)} + I_{k(i)} + \varepsilon_{(ijk)} \tag{7}$$

Source	EMS	Tested By
W_i	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + J\sigma_I^2 + K\sigma_E^2 + JK\sigma_W^2$	$MS(E) + MS(I) - MS(EI)$
$E_{j(i)}$	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + K\sigma_E^2$	$MS(EI)$
$I_{k(i)}$	$\sigma_\varepsilon^2 + \sigma_{EI}^2 + J\sigma_I^2$	$MS(EI)$
$EI_{jk(i)}$	$\sigma_\varepsilon^2 + \sigma_{EI}^2$	−−
$\varepsilon_{(ijk)}$	σ_ε^2	−−

4.5 *Write out the mathematical model for an experiment having both B nested in A, C nested in B, and E nested in D.*

$$\begin{aligned} y_{ijklmn} &= \mu + A_i + B_{j(i)} + C_{k(ij)} + D_l + AD_{il} + BD_{jl(i)} + CD_{kl(ij)} + \\ &\quad E_{m(l)} + AE_{im(l)} + BE_{jm(il)} + CE_{km(ijl)} + \varepsilon_{n(ijklm)} \end{aligned}$$

4.11 *Write out the ANOVA table, including source, df, SS, Δ, EMS, and arrows indicating tests, for problem 4.5. Assume each factor has two levels, A is fixed, B,C,D and E are random, and there are no repeats.*

The ANOVA table has been split up into two parts. The first table consists of source, df, SS and Δ. The second has source, df, EMS and F_{calc} along with the arrows. Notice that no direct test exists for the factors A, B, D, AD, CE and the error term. In order to find Δ, approximate tests were used for all factors with no direct test except CE and the error term where no approximate tests were found. Linear interpolation was used in all cases where appropriate.

Source	df	SS	Δ	Detectable Size
A_i	1	$JKLM\sum(y_{i....}-y_{.....})^2$	2.413	Medium
$B_{j(i)}$	2	$KLM\sum\sum(y_{ij...}-y_{i....})^2$	4.37	Large
$C_{k(ij)}$	4	$LM\sum\sum\sum(y_{ijk..}-y_{ij...})^2$	2.512	Medium
D_l	1	$IJKM\sum(y_{...l.}-y_{.....})^2$	9.487	Extremely Large
AD_{il}	1	$JKM\sum\sum(y_{i..l.}-y_{i....}-y_{...l.}+y_{.....})^2$	13.417	Extremely Large
$BD_{jl(i)}$	2	$KM\sum\sum\sum(y_{ij\cdot l\cdot}-y_{ij...}-y_{i\cdot\cdot l\cdot}+y_{i....})^2$	3.975	Large
$CD_{kl(ij)}$	4	$M\sum\sum\sum\sum(y_{ijkl\cdot}-y_{ijk\cdot\cdot}-y_{ij\cdot k\cdot}+y_{ij...})^2$	1.883	Medium
$E_{m(l)}$	2	$IJK\sum\sum(y_{...lm}-y_{...l\cdot})^2$	4.610	Large
$AE_{im(l)}$	2	$JK\sum\sum\sum(y_{i\cdot\cdot lm}-y_{i\cdot\cdot l\cdot}-y_{...lm}+y_{...l\cdot})^2$	3.193	Large
$BE_{jm(il)}$	4	$IK\sum\sum\sum(y_{\cdot j\cdot lm}-y_{\cdot j\cdot l\cdot}-y_{...lm}+y_{...l\cdot})^2$	3.193	Large
$CE_{km(ijl)}$	8	$\sum\sum\sum\sum\sum(y_{ijklm}-y_{ijkl\cdot}-y_{ij\cdot lm}+y_{ij\cdot l\cdot})^2$	- -	- -

Source	df	EMS	F_{calc}
A_i	1	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+4\sigma^2_{AE}+2\sigma^2_{CD}+4\sigma^2_{BD}+8\sigma^2_{AD}+4\sigma^2_C+8\sigma^2_B+16\sigma^2_A$	$--$
$B_{j(i)}$	2	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+2\sigma^2_{CD}+4\sigma^2_{BD}+4\sigma^2_C+8\sigma^2_B$	$--$
$C_{k(ij)}$	4	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{CD}+4\sigma^2_C$	$MS(C)/MS(CD)$
D_l	1	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+8\sigma^2_E+2\sigma^2_{CD}+4\sigma^2_{BD}+16\sigma^2_D$	$--$
AD_{il}	1	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+4\sigma^2_{AE}+2\sigma^2_{CD}+4\sigma^2_{BD}+8\sigma^2_{AD}$	$--$
$BD_{jl(i)}$	2	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+2\sigma^2_{CD}+4\sigma^2_{BD}$	$--$
$CD_{kl(ij)}$	4	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{CD}$	$MS(CD)/MS(CE)$
$E_{m(l)}$	2	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+8\sigma^2_E$	$MS(E)/MS(BE)$
$AE_{im(l)}$	2	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}+4\sigma^2_{AE}$	$MS(AE)/MS(BE)$
$BE_{jm(il)}$	4	$\sigma^2_\varepsilon+\sigma^2_{CE}+2\sigma^2_{BE}$	$MS(BE)/MS(CE)$
$CE_{km(ijl)}$	8	$\sigma^2_\varepsilon+\sigma^2_{CE}$	$--$
$\varepsilon_{n(ijklm)}$	0	σ^2_ε	$--$

4.16 *What should you do if there is a factor called supplier having two levels, supplier 1 supplies two materials, and supplier 2 supplies three different materials?*

Incorporate supplier and material so that there are now five different levels for the single factor.

4.35 *An experiment was conducted to determine the effect of lead amount, A, trace material amount, T, and location, L, of "buttons" used in maintenance free batteries. Physically, the lead and trace materials are added to a base and poured into a mold, forming many rods. Two buttons are cut off the top (location 1) and bottom (location 2) of three randomly selected rods. Each button is tested for cranking power, higher being better. There are two levels of lead amount, two levels of trace material amount, and the experiment is completely randomized. Give the mathematical model and the ANOVA table including source, df, Δ, EMS and tests.*

The model for this experiment is

$$\begin{aligned} y_{ijklm} &= \mu + A_i + T_j + AT_{ij} + R_{k(ij)} + L_l + \\ & AL_{il} + TL_{jl} + ATL_{ijl} + RL_{kl(ij)} + \varepsilon_{m(ijkl)} \end{aligned} \quad (8)$$

And the ANOVA table with arrows indicating the tests.

Source	*df*	Δ	EMS	F_{calc}
A_i	1	0.758	$\sigma_\varepsilon^2 + 4\sigma_R^2 + 24\Phi(A)$	$MS(A)/MS(R)$
T_j	1	0.758	$\sigma_\varepsilon^2 + 4\sigma_R^2 + 24\Phi(T)$	$MS(T)/MS(R)$
AT_{ij}	1	1.072	$\sigma_\varepsilon^2 + 4\sigma_R^2 + 12\Phi(AT)$	$MS(AT)/MS(R)$
$R_{k(ij)}$	8	1.110	$\sigma_\varepsilon^2 + 4\sigma_R^2$	$MS(R)/MS(\varepsilon)$
L_l	1	0.758	$\sigma_\varepsilon^2 + 2\sigma_{RL}^2 + 24\Phi(L)$	$MS(L)/MS(RL)$
AL_{il}	1	1.072	$\sigma_\varepsilon^2 + 2\sigma_{RL}^2 + 12\Phi(AL)$	$MS(AL)/MS(RL)$
TL_{jl}	1	1.072	$\sigma_\varepsilon^2 + 2\sigma_{RL}^2 + 12\Phi(TL)$	$MS(TL)/MS(RL)$
ATL_{ijl}	1	1.515	$\sigma_\varepsilon^2 + 2\sigma_{RL}^2 + 6\Phi(ATL)$	$MS(ATL)/MS(RL)$
$RL_{kl(ij)}$	8	1.570	$\sigma_\varepsilon^2 + 2\sigma_{RL}^2$	$MS(RL)/MS(\varepsilon)$
$\varepsilon_{m(ijkl)}$	24	$--$	σ_ε^2	$--$

4.36 *The following information was collected in the experiment described in Problem 4.35:* $SS(A) = 1348.00, SS(T) = 175.83, SS(AT) = 335.58, SS(R) = 164.42, SS(L) = 0.25, SS(TL) = 0.34, SS(RL) = 11.01$ *and* $SS(ATL) = 3.00$. *Using our somtimes pooling rules, determine which factors and interactions significantly affect cranking power. Use the following information on means to decide what levels of the factors to use and give a 95% confidence interval on the average cranking power.* $\bar{A}_1 = 158.52, \bar{A}_2 = 147.92, \bar{T}_1 = 151.31$

Source	*df*	SS	MS	F_{calc}
A_i	1	1348.00	1348.00	65.588
T_j	1	175.83	175.83	8.555
AT_{ij}	1	335.58	335.58	16.328
$R_{k(ij)}$	8	164.42	20.5525	0.241
L_l	1	4.19	4.19	3.044
AL_{il}	1	0.25	0.25	0.182
TL_{jl}	1	0.34	0.34	0.247
ATL_{ijl}	1	3.00	3.00	2.180
$RL_{kl(ij)}$	8	11.01	1.37625	0.0161
$\varepsilon_{m(ijkl)}$	24	2042.62	85.109	$--$

I ran out of time to complete this problem.