Chapter 3 Homework

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4. *A factor often used in designs is Weeks. Give a set of circumstances under which Weeks could be considered a fixed factor and under which Weeks could be considered a random factor.*

   The variable Weeks could be considered fixed if an experiment was conducted during each of a specific number of weeks. It would be random if the design included randomly selecting the weeks during which the experiment would be conducted.

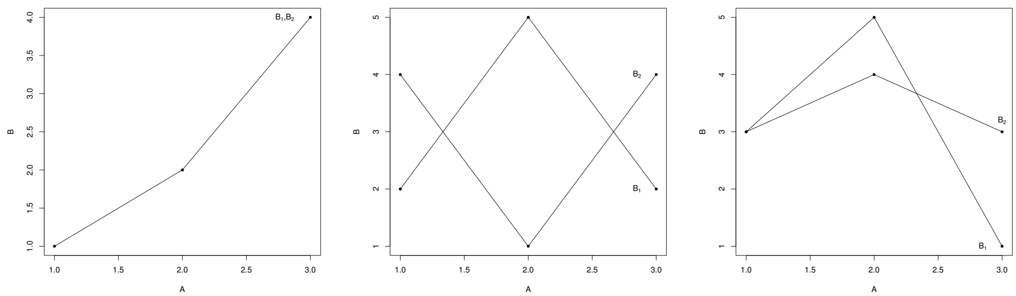
5. *Write the general model for a four factor completely randomized design having three repeats. Use the letters A_i having two levels, B_j having three levels, C_k having four levels and D_l having two levels. How many experimental units are required for this experiment?*

   The general model would be

$$\begin{aligned} y_{ijklm} &= \mu + A_i + B_j + AB_{ij} + C_k + AC_{ik} + BC_{jk} + ABC_{ijk} \\ &\quad + D_l + AD_{il} + BD_{jl} + CD_{kl} + ABD_{ijl} + ACD_{ikl} \\ &\quad + BCD_{jkl} + ABCD_{ijkl} + \varepsilon_{m(ijkl)} \end{aligned}$$

   Subject to the assumptions $A_i \sim N(0, \sigma_A^2)$, $B_j \sim N(0, \sigma_B^2)$, $AB_{ij} \sim N(0, \sigma_{AB}^2)$, $C_k \sim N(0, \sigma_C^2)$, $AC_{ik} \sim N(0, \sigma_{AC}^2)$, $BC_{jk} \sim N(0, \sigma_{BC}^2)$, $ABC_{ijk} \sim N(0, \sigma_{ABC}^2)$, $D_l \sim N(0, \sigma_D^2)$, $AD_{il} \sim N(0, \sigma_{AD}^2)$, $BD_{jl} \sim N(0, \sigma_{BD}^2)$, $CD_{kl} \sim N(0, \sigma_{CD}^2)$, $ABD_{ijl} \sim N(0, \sigma_{ABD}^2)$, $BCD_{ikl} \sim N(0, \sigma_{BCD}^2)$, $BCD_{jkl} \sim N(0, \sigma_{BCD}^2)$, $ABCD_{ijkl} \sim N(0, \sigma_{ABCD}^2)$, $\varepsilon_{m(ijkl)} \sim N(0, \sigma_\varepsilon^2)$. For the factors in our model we have $I = 2$, $J = 3$, $K = 4$, $L = 2$ and $M = 3$. This means that there are $IJKLM = 144$ repetitions of the experiment.

6. *Using A_i fixed at three levels and B_j fixed at two levels, sketch a picture having A_i significant, B_j and AB_{ij} insignificant. Sketch a picture having A_i and B_j insignificant but AB_{ij} significant. Sketch a picture having B_j insignificant, A_i and AB_{ij} significant.*



12. *Use the rules given in Section 3.3 to derive the formulae for df, SS, and MS for the two factor completely randomized design given by (3.2.2)*

    The two factor model for this problem is

$$y_{ijk} = \mu + A_i + B_j + AB_{ij} + \varepsilon_{k(ij)} \tag{1}$$

    And the formulae will be

Source	df	SS	MS
A_i	$I-1$	$JK\sum_i(\bar{y}_{i\cdot\cdot}-\bar{y}_{\cdot\cdot\cdot})^2$	$SS(A)/(I-1)$
B_j	$J-1$	$IK\sum_j(\bar{y}_{\cdot j\cdot}-\bar{y}_{\cdot\cdot\cdot})^2$	$SS(B)/(J-1)$
AB_{ij}	$(I-1)(J-1)$	$K\sum_i\sum_j(\bar{y}_{ij\cdot}-\bar{y}_{i\cdot\cdot}-\bar{y}_{\cdot j\cdot}+\bar{y}_{\cdot\cdot\cdot})^2$	$SS(AB)/(I-1)(J-1)$
$\varepsilon_{k(ij)}$	$IJ(K-1)$	$\sum_i\sum_j\sum_k(\bar{y}_{ijk}-\bar{y}_{ij\cdot})^2$	$SS(\varepsilon)/IJ(K-1)$
Total	$IJK-1$	$\sum_i\sum_j\sum_k(\bar{y}_{ijk}-\bar{y}_{\cdot\cdot\cdot})^2$	

15. *Assuming A_i fixed and B_j random, use the rules in Section 3.4 to derive the EMS values for the model (3.2.2)*

		I/f	J/r	K/r	
Source	df	i	j	k	E(MS)
A_i	$I-1$	0	J	K	$\sigma^2_\varepsilon + K\sigma^2_{AB} + K\sigma^2_B + JK\Phi(A)$
B_j	$J-1$	I	1	K	$\sigma^2_\varepsilon + IK\sigma^2_B$
AB_{ij}	$(I-1)(J-1)$	0	1	K	$\sigma^2_\varepsilon + K\sigma^2_{AB}$
$\varepsilon_{k(ij)}$	$IJ(K-1)$	1	1	1	σ^2_ε

22. *For a three factor experiment with no repeats, derive the exact, approximate, and conservative tests under the following circumstances. When the test is conservative, indicate which conclusion can be made and which conclusion cannot be made.*

    (a) *All three factors are fixed.*

	df	F/i	F/j	F/k	R/l	EMS
A_i	(I-1)	0	J	K	1	$JK\Phi(A)$
B_j	(J-1)	I	0	K	1	$IK\Phi(B)$
AB_{ij}	(I-1)(J-1)	0	0	K	1	$K\Phi(AB)$
C_k	(K-1)	I	J	0	1	$IJ\Phi(C)$
AC_{ik}	(I-1)(K-1)	0	J	0	1	$J\Phi(AC)$
BC_{jk}	(J-1)(K-1)	I	0	0	1	$I\Phi(BC)$
ABC_{ijk}	(I-1)(J-1)(K-1)	0	0	0	1	$\Phi(ABC)$
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

All interactions are conservatively tested by ABC for significance. There is no test for ABC.

(b) *Only the first factor is random.*

	df	R/i	F/j	F/k	R/l	EMS
A_i	(I-1)	1	J	K	1	$JK\sigma^2_A$
B_j	(J-1)	I	0	K	1	$K\sigma^2_{AB} + K\Phi(B)$
AB_{ij}	(I-1)(J-1)	1	0	K	1	$K\sigma^2_{AB}$
C_k	(K-1)	I	J	0	1	$J\sigma^2_{AC} + J\sigma^2_C$
AC_{ik}	(I-1)(K-1)	1	J	0	1	$J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	0	0	1	$\sigma^2_{ABC} + I\Phi(BC)$
ABC_{ijk}	(I-1)(J-1)(K-1)	1	0	0	1	σ^2_{ABC}
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

The table shows that AC tests C, AB tests B and ABC tests BC. We also have conservative tests (for significance) by ABC for A, AB and AC. The second order interaction ABC is not testable.

(c) *Only the first factor is fixed.*

	df	F/i	R/j	R/k	R/l	EMS
A_i	(I-1)	0	J	K	1	$\sigma^2_{ABC} + J\sigma^2_{AC} + K\sigma^2_{AB} + JK\Phi(A)$
B_j	(J-1)	I	1	K	1	$I\sigma^2_{BC} + IK\sigma^2_B$
AB_{ij}	(I-1)(J-1)	0	1	K	1	$\sigma^2_{ABC} + K\sigma^2_{AB}$
C_k	(K-1)	I	J	1	1	$I\sigma^2_{BC} + IJ\sigma^2_C$
AC_{ik}	(I-1)(K-1)	0	J	1	1	$\sigma^2_{ABC} + J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	1	1	1	$I\sigma^2_{BC}$
ABC_{ijk}	(I-1)(J-1)(K-1)	0	1	1	1	σ^2_{ABC}
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

ABC tests AC and AB. BC tests B and C. ABC conservatively tests BC for significance. A is tested approximately with $AB + AC - ABC$.

(d) *All three factors are random.*

	df	R/i	R/j	R/k	R/l	EMS
A_i	(I-1)	1	J	K	1	$\sigma^2_{ABC} + J\sigma^2_{AC} + K\sigma_{AB} + KJ\sigma^2_A$
B_j	(J-1)	I	1	K	1	$\sigma^2_{ABC} + I\sigma^2_{BC} + K\sigma^2_{AB} + IK\sigma^2_B$
AB_{ij}	(I-1)(J-1)	1	1	K	1	$\sigma^2_{ABC} + K\sigma^2_{AB}$
C_k	(K-1)	I	J	1	1	$\sigma^2_{ABC} + I\sigma^2_{BC} + J\sigma^2_{AC} + IJ\sigma^2_C$
AC_{ik}	(I-1)(K-1)	1	J	1	1	$\sigma^2_{ABC} + J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	1	1	1	$\sigma^2_{ABC} + I\sigma^2_{BC}$
ABC_{ijk}	(I-1)(J-1)(K-1)	1	1	1	1	σ^2_{ABC}
$\varepsilon_{l(ijk)}$		1	1	1	0	- -

ABC tests AB, AC and BC. A is approximately tested by $AB + AC - ABC$. B is approximately tested by $AB + BC - ABC$. C is approximately tested by $AC + BC - ABC$. There is no test for ABC.

23. *Repeat problem 3.22 assuming the three factor interaction is zero.*

(a) *All three factors are fixed.*

	df	F/i	F/j	F/k	R/l	*EMS*
A_i	(I-1)	0	J	K	1	$JK\Phi(A)$
B_j	(J-1)	I	0	K	1	$IK\Phi(B)$
AB_{ij}	(I-1)(J-1)	0	0	K	1	$K\Phi(AB)$
C_k	(K-1)	I	J	0	1	$IJ\Phi(C)$
AC_{ik}	(I-1)(K-1)	0	J	0	1	$J\Phi(AC)$
BC_{jk}	(J-1)(K-1)	I	0	0	1	$I\Phi(BC)$
ABC_{ijk}	(I-1)(J-1)(K-1)	0	0	0	1	0
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

All interactions can be tested using ABC directly.

(b) *Only the first factor is random.*

	df	R/i	F/j	F/k	R/l	EMS
A_i	(I-1)	1	J	K	1	$JK\sigma^2_A$
B_j	(J-1)	I	0	K	1	$K\sigma^2_{AB} + K\Phi(B)$
AB_{ij}	(I-1)(J-1)	1	0	K	1	$K\sigma^2_{AB}$
C_k	(K-1)	I	J	0	1	$J\sigma^2_{AC} + J\sigma^2_C$
AC_{ik}	(I-1)(K-1)	1	J	0	1	$J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	0	0	1	$I\Phi(BC)$
ABC_{ijk}	(I-1)(J-1)(K-1)	1	0	0	1	0
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

ABC tests A, AB, AC and BC. AB tests B. AC tests C.

(c) *Only the first factor is fixed.*

	df	F/i	R/j	R/k	R/l	EMS
A_i	(I-1)	0	J	K	1	$J\sigma^2_{AC} + K\sigma^2_{AB} + JK\Phi(A)$
B_j	(J-1)	I	1	K	1	$I\sigma^2_{BC} + IK\sigma^2_B$
AB_{ij}	(I-1)(J-1)	0	1	K	1	$K\sigma^2_{AB}$
C_k	(K-1)	I	J	1	1	$I\sigma^2_{BC} + IJ\sigma^2_C$
AC_{ik}	(I-1)(K-1)	0	J	1	1	$J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	1	1	1	$I\sigma^2_{BC}$
ABC_{ijk}	(I-1)(J-1)(K-1)	0	1	1	1	0
$\varepsilon_{l(ijk)}$	0	1	1	1	0	- -

ABC tests AB, AC and BC. BC tests B and C. A can be approximately tested using $AB + AC - \sigma^2$.

(d) *All three factors are random.*

	df	R/i	R/j	R/k	R/l	EMS
A_i	(I-1)	1	J	K	1	$J\sigma^2_{AC} + K\sigma_{AB} + KJ\sigma^2_A$
B_j	(J-1)	I	1	K	1	$I\sigma^2_{BC} + K\sigma^2_{AB} + IK\sigma^2_B$
AB_{ij}	(I-1)(J-1)	1	1	K	1	$K\sigma^2_{AB}$
C_k	(K-1)	I	J	1	1	$I\sigma^2_{BC} + J\sigma^2_{AC} + IJ\sigma^2_C$
AC_{ik}	(I-1)(K-1)	1	J	1	1	$J\sigma^2_{AC}$
BC_{jk}	(J-1)(K-1)	I	1	1	1	$I\sigma^2_{BC}$
ABC_{ijk}	(I-1)(J-1)(K-1)	1	1	1	1	0
$\varepsilon_{l(ijk)}$		1	1	1	0	- -

ABC tests all first order interactions. A is approximately tested by $AB + AC - \sigma^2$. B is approximately tested by $AB + BC - \sigma^2$. C is approximately tested by $AC + BC - \sigma^2$.