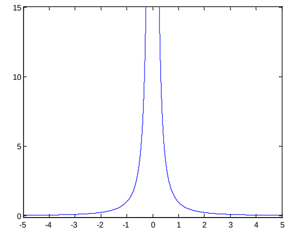
9 Limits Involving Infinity ∞ (2.5)



Consider following limit: $\lim_{x\to 0}\frac{1}{x^2}$. The graph and table (use convenient values like $f\left(10^{-n}\right)=10^{2n}$) suggests that there's no number that the function reach as x approach 0. However one can see that the closer x to the zero (from either side) the larger values the function takes. We'll denote such situation as $\lim_{x\to 0}\frac{1}{x^2}=\infty$. The symbol ∞ doesn't represent a number, but express a particular sort of non-existing limit.

Def: The line x=a called a vertical asymptote of the curve y=f(x) if at least one of the following statements is true: $\lim_{x\to a^\pm}=\infty$, $\lim_{x\to a}=\infty$, $\lim_{x\to a^\pm}=-\infty$, $\lim_{x\to a}=-\infty$

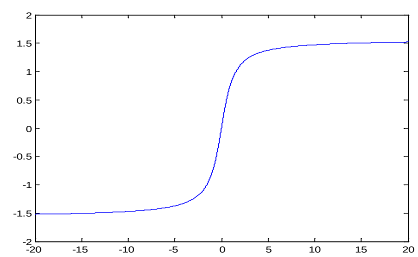
Ex 1. $\lim_{x\to 0}\ln x=-\infty$

Ex 2. $\lim_{x\to \pi/2^-}\tan x=\infty$

Ex 3. $\lim_{x\to -1^+}\frac{x-1}{x^2-1}\cos^2 x=\lim_{x\to -1^+}\frac{x-1}{(x+1)(x-1)}\cos^2 x=\lim_{x\to -1}\frac{1}{x+1^+}\cos^2 x=\infty$ near x=-1 ,

$\cos^2 x<1$ therefore $\frac{\cos^2 x}{x+1}\le\frac{1}{x+1}\to\infty$. Another reason inf*bounded function.

Def: Let f(x) be defined on (a,∞), then $\lim_{x\to\pm\infty}f(x)=L$ means that f(x) gets close to L as x (or –x) gets sufficiently large. We denote the line y=L as horizontal asymptote to y=f(x).



Ex 4. $y=e^{-x}\Rightarrow\lim_{x\to\infty}y=0$

Ex 5. $y=\arctan x\Rightarrow\lim_{x\to\infty}y=\pi/2$

Ex 6. $y=1/x^n\Rightarrow\lim_{x\to\pm\infty}y=0$

Ex 7. a) Let $f(x)=\begin{cases}x & x\in\mathbb{N}\\ 1 & x\in\mathbb{R}\setminus\mathbb{N}\end{cases}$, $\lim_{x\to\infty}f(x)=NA$, $\lim_{\mathbb{N}\ni n\to\infty}f(n)=\infty$, $\lim_{\mathbb{N}\ni n\to\infty}f(n/\pi)=1$

b) Let $g(x)=\begin{cases}1 & x\in\mathbb{N}\\ x^2 & x\in\mathbb{R}\setminus\mathbb{N}\end{cases}$, again $\lim_{x\to\infty}g(x)=NA$, but $\lim_{x\to\infty}f(x)g(x)=\infty$, not that neither f(x) or g(x) tends to infinite, but their multiplication does.

Ex 8. $\lim_{x\to\infty}|x\sin x|=\infty$ since $|\sin x|\le 1\Rightarrow|x\sin x|=|x||\sin x|\le|x|\to\infty$

9.1 The arithmetic of Infinite Limits

1. Consider that $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$ and, then the following is true:

$\lim_{x\to a} \left(f(x) + g(x) \right) = \infty$

but $\lim_{x\to a} \left(f(x) - g(x) \right) = ???$ need more work

$\lim_{x\to a} f(x) g(x) = \infty$

$\lim_{x\to a} f(x)^{g(x)} = \infty$

but $\lim_{x\to a} \frac{f(x)}{g(x)} = ???$ need more work

$\lim_{x\to a} (\pm p) f(x) = \pm\infty, \quad \mathbb{R} \ni p > 0$

$\lim_{x\to a} \left(f(x) \right)^q = \infty, \qquad \mathbb{Q} \ni q > 0$

2. Consider that $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} h(x) = \pm L, \ \mathbb{R} \ni L > 0$, then the following is true:

$\lim_{x\to a} \left(f(x) + h(x) \right) = \infty$

$\lim_{x\to a} f(x) h(x) = \pm\infty$

3. Consider that $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} \tilde{h}(x) = 0$ then

$\lim_{x\to a} f(x) \tilde{h}(x) = ???$ need more work

$\lim_{x\to a} f(x)^{\tilde{h}(x)} = ???$ need more work

but $\lim_{x\to a} \frac{f(x)}{\tilde{h}(x)} = ???$ need more work