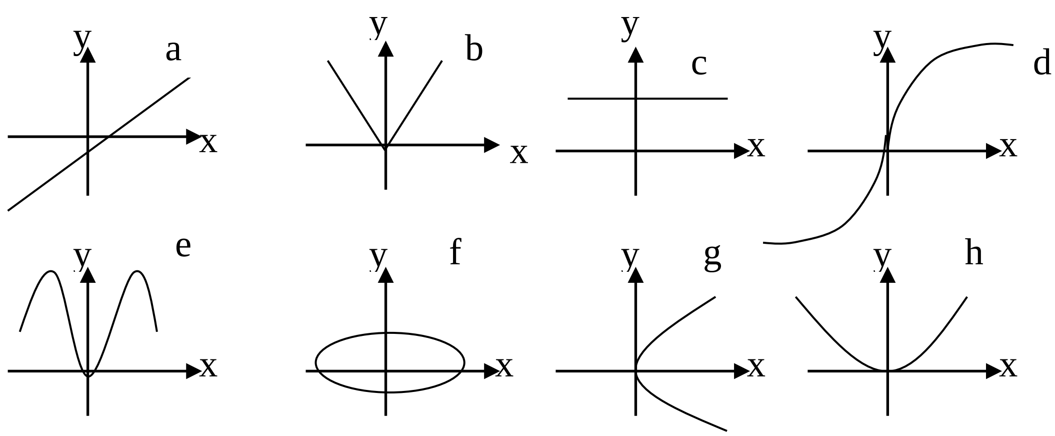
3 Inverse functions (1.6)

Ex 1. Do all graphs below describe a function?



Solution: No, f and g aren't functions, because of multiply values of y for the x.

Note: Function should have only one value for each x in the domain.

Vertical Line Test: A curve in the xy-plane is the graph of function of x if and oly if no vertical line intersects the curve more than once.

DEF: A function is said to be **one-to-one** if it takes different values for any $x \neq y$, i.e. $f(x) \neq f(y)$.

Horizontal Line Test: A one-to-one function is never intersects with horizontal line more than once.

Ex 2. Which function in the graph above is one-to-one?

Solution: a, d

Ex 3. A strictly monotonic (either increasing or decreasing) function is one-to-one.

DEF: A function f:A→B is said to be **onto** (B) if it's range is its image.

Ex 4. a)Which function in the graph above in onto $\mathbb{R}$? b) And which onto $\mathbb{R}^+$?
Solution: a) a and d onto $\mathbb{R}$, b) b and h onto $\mathbb{R}^+$.

DEF: Let **f:A→B** be a function which image is B. A function g:B→A is said to be an inverse function of **f** if for every y=f(x) it gets x=g(y). The relationships between f and it's inverse g are reciprocal\mutual; that is if g is the inverse function of f, then f is the inverse function of g. One denotes inverse function $f^{-1}(x)$.

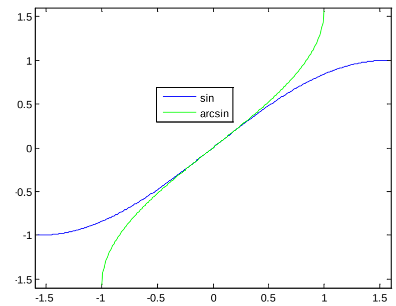
So we have: $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$

NOTE: If a function f is **one-to-one** and **onto** then f^{-1} exists.

NOTE: $f^{-1}(x) \neq \frac{1}{f(x)} = \left[f(x)\right]^{-1}$

Note: The graph of inverse function obtained by reflecting the graph of f about the line x=y.

Ex 5. Graph of sin and arcsin

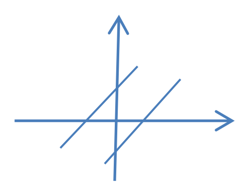


Algorithm: How to find inverse function (of a one-to-one function).

1. Write y=f(x)
2. Solve this equation for x in term of y (if possible)
3. To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$

Ex 6. Find an inverse function of $f(x) = x+1$. Sketch them both.

Solution:



1. $y = x+1$
2. $x = y-1$
3. $y = x-1 = f^{-1}(x)$

Ex 7. Find an inverse function of $f(x) = 2+\sqrt{2x+3}$. Sketch them both.

1. $y = 2+\sqrt{2x+3}$
2. $y-2 = \sqrt{2x+3} \Rightarrow (y-2)^2 = 2x+3 \Rightarrow x = \frac{(y-2)^2-3}{2}$
3. $y = \frac{(x-2)^2-3}{2} = f^{-1}(x)$

Verification: $f\left(f^{-1}(x)\right) = f\left(\frac{(x-2)^2-3}{2}\right) = 2+\sqrt{2\frac{(x-2)^2-3}{2}+3} = 2+\sqrt{(x-2)^2} = 2+x-2 = x$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(2+\sqrt{2x+3}\right) = \frac{\left(2+\sqrt{2x+3}-2\right)^2-3}{2} = \frac{2x+3-3}{2} = x$$