CLOAKING: Where Science meets Science Fiction
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- with thanks to Alexei Efros (Utah) for a key remark

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Becomes invisible by matching his refractive index to that of air

H.G. Wells (1897)
Fantastic Four (1961)

Makes herself invisible by mentally bending all wavelengths of light around her.
Star Trek: Cloaked Romulan Bird of Prey
Camouflage
The art of Beverly Doolittle
Camouflage Artist
Liu Bolin
Camouflage by mirrors
Choi and Howell (2014)
Invisibility cloak of Tachi and collaborators
Transparent Cockpit
Invisible Trains ?
http://www.saferplane.com/stealth/
BAE ADAPTIV Camouflage
Mirages:

Aliev et. al. (2011)
Neutral Inclusions: Invisibility to some fields.
with S. Serkov (2001)
Cloak making an object “unfeelable”: Buckmann et. al. (2014)
Pentamodes blueprint, with Cherkaev 1995 realized: Kadic et.al. 2012
Multicoated cyclinders and spheres can be neutral to multipole fields up to a given order

Conductivity: Ammari et. al. (2012)
Helmholtz: Ammari et. al. (2012)
Maxwell: Ammari et. al. (2013)
The simplest type of cloaking: Cloaking for conductivity
Electrical Impedance Tomography: Cheney et. al. (1999)
Transformation based cloaking


(A) Stretching space

(B) (From Ulf Leonhardt)
Transformation optics discovered: Dolin 1961

TO THE POSSIBILITY OF COMPARISON OF THREE-DIMENSIONAL ELECTROMAGNETIC SYSTEMS WITH NONUNIFORM ANISOTROPIC FILLING

L. S. Dolin

It was shown that it is possible to investigate three-dimensional systems with nonuniform anisotropic filling by comparison them with other, more simple three-dimensional systems. The examination is made basing on an invariance of Maxwells equations relative to the certain type of transformation of space metric and medium permeability and permittivity.

READ THE ABSTRACT ABOVE. IT IS EXACTLY TRANSFORMATION OPTICS.

\[ R(r) \rightarrow r. \]  \hspace{1cm} (4)

\[ \| \varepsilon_{ik} \| = \| \psi_{ik} \| = \begin{vmatrix} \frac{R^2}{r(R)} \frac{dr(R)}{dR} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{dr(R)/dR} \end{vmatrix}. \] \hspace{1cm} (5)

Never cited by Pendry, as far as I can see.
Cloaking for Waves
Putting an object in the orange cloaking region is equivalent to disturbing the medium inside a point in the equivalent problem, which will not disturb the surrounding fields, in particular outside the blue shell where the fields are the same.
Approximate cloak in 2-dimensions

\[ \varepsilon_z = \left( \frac{b}{b-a} \right)^2 \mu_r = \left( \frac{r-a}{r} \right)^2 \mu_\theta = 1 \]
Approximate non-magnetic quadratic cloak with minimized reflectance

\[ \varepsilon_r = \left( \frac{r'}{r} \right)^2; \quad \varepsilon_\theta = \left[ p (2r' - b) + 1 - a/b \right]^2; \quad \mu_z = 1. \quad p = a/b^2 \]
Scattering for the different approximate cloaks
Cloaking of water waves:
Farhat et. al. (2008)
Acoustic cloaking: Zhang, Zia, Fang (2011)
A different type of cloak

Hiding under the carpet: Li and Pendry (2008)
A different type of cloak

Hiding under the carpet: Li and Pendry (2008)

Experimental realization of Valentine et.al. 2009
700nm = red light
Gabrielli et. al. (2009)

Ergin et. al. (2010)
Calcite Cloaking

Zhang. et.al. (2011)
Chen et.al. (2011)
Unidirectional Cloak: Landy and Smith (2013)
Other transformation based cloaking results
Non Euclidean cloaking: Leonhardt and Tyc (2008)
Optical Wormhole: Greenleaf et.al. 2007
An important parallel:

Maxwell’s Equations:

\[
\frac{\partial}{\partial x_i} \left( C_{ijkl} \frac{\partial E_l}{\partial x_k} \right) = \{ \omega^2 \varepsilon E \}_j
\]

\[
C_{ijkl} = e_{ijm} e_{kln} \{ \mu^{-1} \}_{mn}
\]

Continuum Elastodynamics:

\[
\frac{\partial}{\partial x_i} \left( C_{ijkl} \frac{\partial u_l}{\partial x_k} \right) = -\{ \omega^2 \rho u \}_j
\]
Cloaking for elasticity (with Marc Briane and John Willis, 2006)

Idea: Apply the Greenleaf, Lassas and Uhlmann/Pendry, Schurig, Smith method of cloaking to elastic waves

Requires new materials with new behavior, in particular, materials with anisotropic density

Cummer & Schurig (2007), Greenleaf et.al. (2007) Chang & Chan (2007) found that materials with anisotropic density are also needed for cloaking against sound
A material with anisotropic effective density
Transformation of the elastodynamic equations

\[ \nabla \cdot \sigma = -\omega^2 \rho u, \quad \sigma = C \nabla u \]

under the transformation \( x \rightarrow x'(x), \quad u \rightarrow u'(x') \)

with \( u'(x') = (A^T)^{-1} u(x), \quad A_{ij} = \frac{\partial x'_i}{\partial x_j} \)

transform to

\[ \nabla \cdot \sigma' = D' \nabla u' - \omega^2 \rho' u', \quad \sigma = C' \nabla u' + S' u' \]

where

\[ \rho'_{pq} \approx \frac{\rho}{a} \frac{\partial x'_p}{\partial x_i} \frac{\partial x'_q}{\partial x_i} + \frac{1}{\omega^2 a} \frac{\partial^2 x'_p}{\partial x_i \partial x_j} C_{ijk\ell} \frac{\partial^2 x'_q}{\partial x_k \partial x_\ell} \]

\[ C'_{pqrs} \approx \frac{1}{a} \frac{\partial x'_p}{\partial x_i} \frac{\partial x'_q}{\partial x_j} C_{ijk\ell} \frac{\partial x'_r}{\partial x_k} \frac{\partial x'_s}{\partial x_\ell} \]

\[ S'_{pqr} = \frac{1}{a} \frac{\partial x'_p}{\partial x_i} \frac{\partial x'_q}{\partial x_j} C_{ijk\ell} \frac{\partial^2 x'_r}{\partial x_k \partial x_\ell} = S'_{pqr} \]

\[ D'_{pqr} = S'_{qrp} \quad a = \det A \]
These equations don’t look like elastodynamic equations at all! But they are a special case of the Willis equations

\[
\begin{pmatrix}
    \langle \sigma \rangle \\
    \langle p \rangle
\end{pmatrix}
= 
\begin{pmatrix}
    C^{\text{eff}} & S^{\text{eff}} \\
    (S^{\text{eff}})^\dagger & \rho^{\text{eff}}
\end{pmatrix}
\ast
\begin{pmatrix}
    \langle \epsilon \rangle \\
    \langle \dot{u} \rangle
\end{pmatrix}
\]

\[\nabla \cdot \langle \sigma \rangle = \langle \dot{p} \rangle, \quad \langle \epsilon \rangle = [\nabla \langle u \rangle + (\nabla \langle u \rangle)^T]/2
\]

Brackets \( \langle \rangle \) denote ensemble averages

\ast \text{ denotes a convolution with respect to time}

\( C^{\text{eff}}, S^{\text{eff}}, \rho^{\text{eff}} \) are operators which are generally non-local in space
However unlike the Willis equations, one wants the constitutive law to be local in space and to apply (macroscopically) to a single realization of a microstructure rather than the ensemble average.

Are there composites where such a constitutive law is realized?

Note: Brun, Guenneau and Movchan (2009) have shown that one can obtain cloaks with $S=D=0$ with $C$ not satisfying the usual symmetries, by letting $u'(x') = u(x)$
Sheng, Zhang, Liu, and Chan (2003) found that materials could exhibit a negative effective density over a range of frequencies.

Red=Rubber, Black=Lead, Blue=Stiff Matrix
The Black circles have positive effective mass.
The White circles have negative effective mass.
Another idea for cloaking of elastic waves in plates was suggested by Farhat et.al. (2009) and experimentally realized by Stenger et.al. (2012)
Cloaking on a grand scale: seismic cloaking
Brule et.al. (2014)

Sensitive three components velocimeters (green grid)
Five meters deep 320 mm holes
Source:
- Frequency: 50 Hz
- Horizontal displacement: 14 mm
Exterior Cloaking.

Cloaking at a distance!
Besides transformation based cloaking there is also cloaking due to anomalous resonance which we discovered in 2005, prior to the work of Pendry, Schurig and Smith and Leonhardt on transformation based cloaking.

In contrast to transformation based cloaking the cloaking region lies outside the cloaking device.
\[ \nabla \cdot \epsilon \nabla V = 0 \]

Is equivalent to

\[ r_* = r_s^2 / r_c \]

\[ h = r_s^2 / r_c^2 \]

How and in what sense can the equivalence hold?
Ghost sources and anomalous resonance are the essential mechanisms that explain superlensing.
Later Simulation

\[ \varepsilon_c = 100 \]
\[ \varepsilon_s = -1 + 10^{-12} \varepsilon \]
\[ \varepsilon_m = 1 \]
When the shell was hollow we found it was completely invisible to any applied field.
Very similar phenomena were later found to be associated with the Veselago lens by Pendry (2000) and subsequent workers. Their work again indicated apparent point image sources in the physical region and large fields on one side of them.
Negative Refraction Simulation: Hess 2008
Negative refraction at optical frequencies: Valentine et al. (2008)
The superlens: Pendry (2000)

Veselago Lens (1967)

Wrong Picture
Folding space: Leonhardt and Philbin (2006)

Wrong Picture

Numerical Results of Cummer (2003) showing the anomalously resonant regions on both sides of the lens.
Numerical results of Shvets (2003) indicating the anomalously resonant regions centered at both the front and back sides of the lens.
Finite power absorbed

Non-overlapping resonant regions

Infinite power absorbed!!

Overlapping resonant regions
When the source is closer than a distance \( d/2 \) from the lens it interacts with the enormously large anomalously resonant fields in front of the lens.

These fields act like a sort of “optical molasses” against which the point source has to do work per unit time, in fact an infinite amount of work per unit time as the loss in the lens goes to zero.

Any realistic point source can only supply a finite amount of energy per unit time, and therefore its amplitude must go to zero. It will be cloaked!
Number of Citations does not establish validity

Negative Refraction Makes a Perfect Lens

J. B. Pendry

Condensed Matter Theory Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
(Received 25 April 2000)

10,000 citations but wrong when the source is near the lens

Opaque perfect lenses

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11 citations, excluding self-citations and correct
$k_1(e/o) = -0.90585, -0.11345$

$k_2(e/o) = -0.79466, 0.44939$
Solutions in folded geometries, and associated cloaking due to anomalous resonance

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Focus on Cloaking and Transformation Optics

\[ \varepsilon_s = -1 + 10^{-9}i \]

\[ \varepsilon_s = -1 + 10^{-15}i. \]
Complimentary media cloaking: Lai et. al. (2009)
Broadband Active Cloaking

with Fernando Guevara Vasquez and Daniel Onofrei
Uncloaked  

Cloaked
Green’s formula cloak of Miller (2006)

\[
  u_d(x) = \int_{\partial D} dS_y \left\{ -\mathbf{n}(y) \cdot \nabla_y u_i(y) \right\} G(x, y) + u_i(y) \mathbf{n}(y) \cdot \nabla_y G(x, y) \right\},
\]

\[
  G(x, y) = \frac{i}{4} H_0^{(1)}(k |x - y|)
\]
Experimental realization of active cloaking: Selvanayagam and Eleftheriades (2013)
Use the addition formula:
Cloaking for 3-dimensional acoustics

\[(b) \ |u_d| = 5\]
Thank you!

Thank you!
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