

1. Find all isolated singularities of the function

$$f(z) = \frac{z}{e^z - 1}$$

and determine their type.

2. Find all isolated singularities of the function

$$f(z) = \sin\left(\frac{z}{z+1}\right)$$

and determine their type.

3. Using the residue theorem, find

$$\int_0^{2\pi} \frac{d\varphi}{(a + b \cos \varphi)^2}$$

where $a > b > 0$.

4. Using the residue theorem, find

$$\int_{-\infty}^{\infty} \frac{x dx}{(x^2 + 4x + 13)^2}$$

5. Using the residue theorem, find

$$\int_{-\infty}^{\infty} \frac{x \cos x dx}{x^2 - 2x + 10}$$

6. Using the residue theorem, find

$$\int_0^{\infty} \frac{x^{-p} dx}{x^2 + 2x \cos \lambda + 1}$$

where $-1 < p < 1$ and $-\pi < \lambda < \pi$.

7. Let f be an entire function with a pole at ∞ . Show that f is a polynomial.
8. Let f be an entire function which is a bijection of the complex plane \mathbb{C} onto itself. Show that f is a nonconstant linear function.
9. How many zeros of the polynomial $P(z) = z^9 - 2z^6 + z^2 - 8z - 2$ lie inside unit circle?
10. Let f be a function holomorphic in a neighborhood of the closed unit disk. Assume that it maps this closed disk into its interior. How many fixed points does f have in the disk?