Problem 1. Let \( f \) be a periodic continuous function on \( \mathbb{R} \) with period \( 2\pi \). Denote by
\[
\sum_{n \in \mathbb{Z}} c_n e^{inx}
\]
its Fourier series. Show that the following conditions are equivalent:
(i) the function \( f \) is infinitely differentiable;
(ii) for any \( k \geq 0 \) there exists \( M > 0 \) such that \(|c_n||n|^k \leq M\) for all \( n \in \mathbb{Z} \).
(Hint: Use Theorem 7.17 from Rudin!)

Problem 2. Let \( A \) be a linear map from \( \mathbb{R}^n \) into \( \mathbb{R} \). Show that
(i) there is a unique vector \( y \in \mathbb{R}^n \) such that \( A(x) = (x \mid y) \) for all \( x \in \mathbb{R}^n \);
(ii) \( \|A\| = |y| \).

Problem 3. If \( F \) is a differentiable real function defined in a convex open set \( E \subset \mathbb{R}^n \), such that \( \partial_1 F(x) = 0 \) for every \( x \in E \), prove that \( F \) depends only on \( x_2, \ldots, x_n \).

Problem 4. Let \( f \) be a function on \( \mathbb{R}^2 \) defined by
\[
f(x, y) = \begin{cases} 
0 & \text{if } (x, y) = (0, 0); \\
x y & \text{if } (x, y) \neq (0, 0).
\end{cases}
\]
Prove
(i) \( f \) is not continuous at \( 0 \);
(ii) The first partial derivatives of \( f \) exist at every point of \( \mathbb{R}^2 \).
Is \( f \) differentiable at \( (0, 0) \)?