# Differentiable manifolds

$M$ - topological space

$c = (U, \varphi, n)$

$U$ - open set in $M$

$\varphi : U \longrightarrow \mathbb{R}^n$ homeomorphism

of $U$ onto open set $\varphi(U) \subset \mathbb{R}^n$

is a <u>chart</u> on $M$

$U$ is the <u>domain</u> of $c$

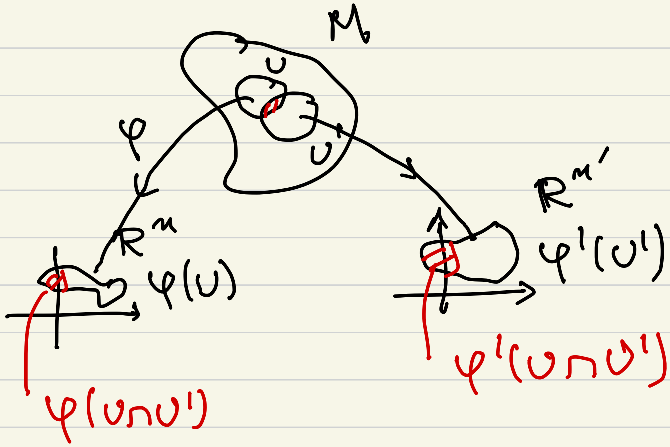
$n$ - <u>dimension</u> of $c$

Two charts $c = (U, \varphi, n)$ and

$c' = (U', \varphi', n')$ are <u>compatible</u>

if either $U \cap U' = \emptyset$ or

$U \cap U' \neq \emptyset$ and




and the map

$$\varphi' \circ \varphi^{-1} : \varphi(U \cap U') \longrightarrow \varphi'(U \cap U')$$

is $C^\infty$ - diffeomorphism.

An <u>atlas</u> on M is a collection of compatible charts on M such that their domains are an open cover of M.

Two atlases $\mathcal{A}$ and $\mathcal{B}$ on M are <u>compatible</u> if their union is an atlas on $M$.

Compatibility of atlases is an equivalence relation. Each equivalence class contains a largest element – the union of all atlases in this class. This is a *saturated atlas*.

A *differentiable manifold* is a hausdorff topological space with a saturated atlas.

A differentiable manifold is a locally compact space (each point has a compact neighborhood).

Variants: ① Assume that

$$\varphi' \circ \varphi^{-1} : \varphi(U \cap U') \longrightarrow \varphi'(U \cap U')$$

is an analytic diffeomorphism (i.e functions are locally given by convergent power series).

– analytic manifold.

② Replace $\mathbb{R}$ with $\mathbb{C}$ and assume that $\varphi' \circ \varphi^{-1}$ are holomorphic diffeomorphisms (diff. in complex sense) – complex manifold.

Let $M, N$ be two manifolds $F : M \longrightarrow N$ a continuous map. $F$ is a differentiable

<u>map</u> if for any pair of charts $c = (U, \varphi, m)$ on $M$ and $d = (V, \psi, n)$ on $N$ such that $F(U) \subset V$ the map

$$\psi \circ F \circ \varphi^{-1} : \varphi(U) \longrightarrow \psi(V)$$

is a $C^\infty$-map (inf. differentiable)..

The <u>category of differentiable manifolds</u> is the category with objects which are differentiable manifolds and morphisms which are differentiable maps.

<u>Examples</u>. $\mathbb{R}$ - real line

$c = (\mathbb{R}, id, 1)$ chart on $\mathbb{R}$

Defines a structure of diff.
manifold on $\mathbb{R}$.

A differentiable function $f$
on a manifold $M$ is a
differentiable map $f : M \longrightarrow \mathbb{R}$,

$C^\infty(M)$ – all differentiable
functions on $M$
– this is an algebra,