Problem 1 (20 points). Determine if the series
\[ \sum_{k=1}^{\infty} \frac{1}{2^k + k - 1} \]
converges or diverges.

Problem 2 (20 points). Prove that the function
\[ f(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{2^k} \]
is defined and continuous on the entire real line.

Problem 3 (20 points). Find the radius of convergence of the series
\[ \sum_{k=0}^{\infty} 2^k x^{2k}. \]

Problem 4 (20 points). Find a power series on \((-1, 1)\) which converges to
\[ \frac{1}{(1 - x)^3}. \]

Problem 5 (20 points). The function *hyperbolic cosine* is defined as
\[ \cosh x = \frac{e^x + e^{-x}}{2}, \]
find its Taylor series for \(a = 0\). Show that the series converges to the function on the entire real line.