Math 3210-02: Homework 6, November 25, 2019

Show all work!

Name:

Problem 1 (20 points)**.** Determine if the series

$$\sum_{k=1}^{\infty} \frac{1}{2^k + k - 1}$$

converges or diverges.

Problem 2 (20 points)**.** Prove that the function

$$f(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{2^k}$$

is defined and continuous on the entire real line.

Problem 3 (20 points)**.** Find the radius of convergence of the series

$$\sum_{k=0}^{\infty} 2^k x^{2k}.$$

Problem 4 (20 points)**.** Find a power series on $(-1, 1)$ which converges to

$$\frac{1}{(1-x)^3}.$$

Problem 5 (20 points)**.** The function *hyperbolic cosine* is defined as

$$\cosh x = \frac{e^x + e^{-x}}{2},$$

find its Taylor series for $a = 0$. Show that the series converges to the function on the entire real line.