Math 3210-02: Homework 5, November 8, 2019

Show all work!

Name:

Problem 1 (20 points)**.** Prove that

$$1 \leq \int_{-1}^{1} \frac{1}{1+x^{2n}}\, dx \leq 2$$

for all $n \in \mathbb{N}$.

Problem 2 (20 points)**.** Let $\{f_n\}$ be a sequence of integrable functions defined on a closed bounded interval $[a, b]$. If $\{f_n\}$ converges uniformly on $[a, b]$ to a function f, prove that f is integrable and

$$\int_a^b f(x)\, dx = \lim \int_a^b f_n(x)\, dx.$$

Problem 3 (20 points)**.** Find

$$\frac{d}{dx} \int_0^{2x} \sin t^2\, dt.$$

Problem 4 (20 points)**.** Let f be a continuous function on the interval $[0, 1]$. Express

$$\int_0^{\frac{\pi}{2}} f(\sin\theta) \cos\theta\, d\theta$$

as an integral involving only the function f.

Problem 5 (20 points)**.** Prove that

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

for all $a, b \in (0, +\infty)$.