Problem 1 (20 points). Using induction prove that

$$\sum_{k=1}^{n} (2k - 1) = n^2$$

for every $n \in \mathbb{N}$.

Problem 2 (20 points). Let $F$ be an ordered field. Show that $x < 0$ implies that $x^{-1} < 0$.

Problem 3 (20 points). Prove that every rational solution of a polynomial equation

$$x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0 = 0$$

with integer coefficients $a_n$ is an integer.

Problem 4 (20 points). Prove that

$$\lim_{n \to \infty} \frac{\sin(n)}{n} = 0.$$

Problem 5 (20 points). Does the sequence $(\sin(n))$ have a convergent subsequence? Why?