EXTREME POINTS OF THE UNIT BALL IN THE DUAL SPACES OF SOME REAL SUBSPACES OF $C(X)$

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Abstract. Let A be a nonzero subspace of a normed space C over the real or the complex numbers. If ϕ belongs to the closed unit ball $L(A)$ of the dual of A, then conditions are given for ϕ to be an extreme point of $L(A)$ in terms of Hahn-Banach extensions of ϕ to C and the extreme points of C. These consideration are applied to the case where $C = C(X)_r$, the normed space of all complex-valued continuous functions on X regarded as a real linear space. In particular, we consider a continuous involution τ on X, let

$$C(X,\tau) = \{f \in C(X)_r : \ f \circ \tau = \bar{f}\}$$

and treat (real) subspace of $C(X,\tau)$. A characterization of the extreme points of $L(C(X,\tau))$ (given by Grzesiak) is deduced. If A is a real subspace of $C(X,\tau)$ and $\phi \in L(A)$, then the uniqueness of the representing measures for ϕ is related to ϕ being an extreme point of $L(A)$.

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