2. (a) The average velocity over \(2 \leq t \leq 2.4\) is
\[
\frac{\mathbf{r}(2.4) - \mathbf{r}(2)}{2.4 - 2} = 2.5 \left[ \mathbf{r}(2.4) - \mathbf{r}(2) \right],
\]
so we sketch a vector in the same direction but 2.5 times the length of \(\left[ \mathbf{r}(2.4) - \mathbf{r}(2) \right]\).

(b) The average velocity over \(1.5 \leq t \leq 2\) is
\[
\frac{\mathbf{r}(2) - \mathbf{r}(1.5)}{2 - 1.5} = 2[\mathbf{r}(2) - \mathbf{r}(1.5)],
\]
so we sketch a vector in the same direction but twice the length of \(\left[ \mathbf{r}(2) - \mathbf{r}(1.5) \right]\).

(c) Using Equation 2 we have \(\mathbf{v}(2) = \lim_{h \to 0} \frac{\mathbf{r}(2 + h) - \mathbf{r}(2)}{h}\).

(d) \(\mathbf{v}(2)\) is tangent to the curve at \(\mathbf{r}(2)\) and points in the direction of increasing \(t\). Its length is the speed of the particle at \(t = 2\). We can estimate the speed by averaging the lengths of the vectors found in parts (a) and (b) which represent the average speed over \(2 \leq t \leq 2.4\) and \(1.5 \leq t \leq 2\) respectively. Using the axes scale as a guide, we estimate the vectors to have lengths 2.8 and 2.7. Thus, we estimate the speed at \(t = 2\) to be \(|\mathbf{v}(2)| \approx \frac{1}{2} (2.8 + 2.7) = 2.75\) and we draw the velocity vector \(\mathbf{v}(2)\) with this length.
14. \( \mathbf{a}(t) = 2\mathbf{i} + 6t\mathbf{j} + 12t^2\mathbf{k} \Rightarrow \mathbf{v}(t) = \int (2\mathbf{i} + 6t\mathbf{j} + 12t^2\mathbf{k}) \, dt = 2t\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k} + \mathbf{C} \), and \( \mathbf{i} = \mathbf{v}(0) = \mathbf{C} \), so \( \mathbf{C} = \mathbf{i} \) and \( \mathbf{v}(t) = (2t + 1)\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k} \). 

\[ \mathbf{r}(t) = \int [(2t+1)\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k}] \, dt = (t^2 + t)\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k} + \mathbf{D}. \]

But \( \mathbf{j} - \mathbf{k} = \mathbf{r}(0) = \mathbf{D} \), so \( \mathbf{D} = \mathbf{j} - \mathbf{k} \) and \( \mathbf{r}(t) = (t^2 + t)\mathbf{i} + (t^3 + 1)\mathbf{j} + (t^4 - 1)\mathbf{k} \).
13. \( r(u, v) = u \cos v \, \mathbf{i} + u \sin v \, \mathbf{j} + v \, \mathbf{k} \). The parametric equations for the surface are \( x = u \cos v \), \( y = u \sin v \), \( z = v \). We look at the grid curves first; if we fix \( v \), then \( x \) and \( y \) parametrize a straight line in the plane \( z = v \) which intersects the \( z \)-axis. If \( u \) is held constant, the projection onto the \( xy \)-plane is circular; with \( z = v \), each grid curve is a helix. The surface is a spiraling ramp, graph IV.

14. \( r(u, v) = u \cos v \, \mathbf{i} + u \sin v \, \mathbf{j} + \sin u \, \mathbf{k} \). The corresponding parametric equations for the surface are \( x = u \cos v \), \( y = u \sin v \), \( z = \sin u \). If \( u = u_0 \) is held constant, then \( x = u_0 \cos v \), \( y = u_0 \sin v \) so each grid curve is a circle of radius \( |u_0| \) in the horizontal plane \( z = \sin u_0 \). If \( v = v_0 \) is constant, then \( x = u \cos v_0 \), \( y = u \sin v_0 \) \( \Rightarrow \ y = (\tan v_0) x \), so the grid curves lie in vertical planes \( y = kx \) through the \( z \)-axis. In fact, since \( x \) and \( y \) are constant multiples of \( u \) and \( z = \sin u \), each of these traces is a sine wave. The surface is graph I.

15. \( r(u, v) = \sin v \, \mathbf{i} + \cos u \, \sin 2v \, \mathbf{j} + \sin u \, \sin 2v \, \mathbf{k} \). Parametric equations for the surface are \( x = \sin v \), \( y = \cos u \sin 2v \), \( z = \sin u \sin 2v \). If \( v = v_0 \) is fixed, then \( x = \sin v_0 \) is constant, and \( y = (\sin 2v_0) \cos u \) and \( z = (\sin 2v_0) \sin u \) describe a circle of radius \( |\sin 2v_0| \), so each corresponding grid curve is a circle contained in the vertical plane \( x = \sin v_0 \) parallel to the \( yz \)-plane. The only possible surface is graph II. The grid curves we see running lengthwise along the surface correspond to holding \( u \) constant, in which case \( y = (\cos u_0) \sin 2v \), \( z = (\sin u_0) \sin 2v \) \( \Rightarrow \ z = (\tan u_0) y \), so each grid curve lies in a plane \( z = ky \) that includes the \( x \)-axis.
16. \( x = (1 - u)(3 + \cos v) \cos 4\pi u, \ y = (1 - u)(3 + \cos v) \sin 4\pi u, \ z = 3u + (1 - u) \sin v. \) These equations correspond to graph V: when \( u = 0, \) then \( x = 3 + \cos v, \ y = 0, \) and \( z = \sin v, \) which are equations of a circle with radius 1 in the \( xz-plane \) centered at \((3, 0, 0)\). When \( u = \frac{1}{2}, \) then \( x = \frac{3}{2} + \frac{1}{2} \cos v, \ y = 0, \) and \( z = \frac{3}{2} + \frac{1}{2} \sin v, \) which are equations of a circle with radius \( \frac{1}{2} \) in the \( xz-plane \) centered at \((\frac{3}{2}, 0, \frac{3}{2})\). When \( u = 1, \) then \( x = y = 0 \) and \( z = 3, \) giving the topmost point shown in the graph. This suggests that the grid curves with \( u \) constant are the vertically oriented circles visible on the surface. The spiralling grid curves correspond to keeping \( v \) constant.

17. \( x = \cos^3 u \cos^3 v, \ y = \sin^3 u \cos^3 v, \ z = \sin^3 v. \) If \( v = v_0 \) is held constant then \( z = \sin^3 v_0 \) is constant, so the corresponding grid curve lies in a horizontal plane. Several of the graphs exhibit horizontal grid curves, but the curves for this surface are neither circles nor straight lines, so graph III is the only possibility. (In fact, the horizontal grid curves here are members of the family \( x = a \cos^3 u, \ y = a \sin^3 u \) and are called astroids.) The vertical grid curves we see on the surface correspond to \( u = u_0 \) held constant, as then we have \( x = \cos^3 u_0 \cos^3 v, \ y = \sin^3 u_0 \cos^3 v \) so the corresponding grid curve lies in the vertical plane \( y = (\tan^3 u_0)x \) through the \( z \)-axis.

18. \( x = (1 - |u|) \cos v, \ y = (1 - |u|) \sin v, \ z = u. \) Then \( x^2 + y^2 = (1 - |u|)^2 \cos^2 v + (1 - |u|)^2 \sin^2 v = (1 - |u|)^2, \) so if \( u \) is held constant, each grid curve is a circle of radius \((1 - |u|)\) in the horizontal plane \( z = u. \) The graph then must be graph VI. If \( v \) is held constant, so \( v = v_0, \) we have \( x = (1 - |u|) \cos v_0 \) and \( y = (1 - |u|) \sin v_0. \) Then \( y = (\tan v_0)x, \) so the grid curves we see running vertically along the surface in the planes \( y = kx \) correspond to keeping \( v \) constant.
26. Using $x$ and $y$ as the parameters, $x = x$, $y = y$, $z = x + 3$ where $0 \leq x^2 + y^2 \leq 1$. Also, since the plane intersects the cylinder in an ellipse, the surface is a planar ellipse in the plane $z = x + 3$. Thus, parametrizing with respect to $s$ and $\theta$, we have $x = s \cos \theta$, $y = s \sin \theta$, $z = 3 + s \cos \theta$ where $0 \leq s \leq 1$ and $0 \leq \theta \leq 2\pi$. 
30. Letting $\theta$ be the angle of rotation about the $y$-axis, we have the parametrization $x = (4y^2 - y^4) \cos \theta$, $y = y$, $z = (4y^2 - y^4) \sin \theta$, $-2 \leq y \leq 2$, $0 \leq \theta \leq 2\pi$. 
10. (a) \( C \) (Chicago) lies between level curves with pressures 1012 and 1016 mb, and since \( C \) appears to be located about one-fourth the distance from the 1012 mb isobar to the 1016 mb isobar, we estimate the pressure at Chicago to be about 1013 mb. \( N \) lies very close to a level curve with pressure 1012 mb so we estimate the pressure at Nashville to be approximately 1012 mb. \( S \) appears to be just about halfway between level curves with pressures 1008 and 1012 mb, so we estimate the pressure at San Francisco to be about 1010 mb. \( V \) lies close to a level curve with pressure 1016 mb but we can't see a level curve to its left so it is more difficult to make an accurate estimate. There are lower pressures to the right of \( V \) and \( V \) is a short distance to the left of the level curve with pressure 1016 mb, so we might estimate that the pressure at Vancouver is about 1017 mb.

(b) Winds are stronger where the isobars are closer together (see Figure 6), and the level curves are closer near \( S \) than at the other locations, so the winds were strongest at San Francisco.
20. The level curves are \( x^3 - y = k \) or \( y = x^3 - k \), a family of cubic curves.
10. \( f(x, y) = \frac{6x^3y}{(2x^4 + y^4)} \). On the \( x \)-axis, \( f(x, 0) = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \) as \( (x, y) \to (0, 0) \) along the \( x \)-axis.

Approaching \( (0, 0) \) along the line \( y = x \) gives \( f(x, x) = \frac{6x^4}{(3x^4)} = 2 \) for \( x \neq 0 \), so along this line \( f(x, y) \to 2 \) as \( (x, y) \to (0, 0) \). Thus the limit does not exist.
12. We can use the Squeeze Theorem to show that \( \lim_{{(x,y) \to (0,0)}} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0:\)

\[
0 \leq \frac{x^2 \sin^2 y}{x^2 + 2y^2} \leq \sin^2 y \quad \text{since} \quad \frac{x^2}{x^2 + 2y^2} \leq 1, \quad \text{and} \quad \sin^2 y \to 0 \quad \text{as} \quad (x,y) \to (0,0),
\]

so

\[
\lim_{{(x,y) \to (0,0)}} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0.
\]
14. \( f(x, y) = \frac{xy^4}{(x^2 + y^8)} \). On the x-axis, \( f(x, 0) = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \) as \( (x, y) \to (0, 0) \) along the x-axis.

Approaching \( (0, 0) \) along the curve \( x = y^4 \) gives \( f(y^4, y) = y^8 / 2y^8 = \frac{1}{2} \) for \( y \neq 0 \), so along this path \( f(x, y) \to \frac{1}{2} \) as \( (x, y) \to (0, 0) \). Thus the limit does not exist.
28. $F(x, y) = \cos \sqrt{1 + x - y} = g(f(x, y))$ where $f(x, y) = \sqrt{1 + x - y}$, continuous on its domain

\[ \{(x, y) \mid 1 + x - y \geq 0\} = \{(x, y) \mid y \leq x + 1\} \text{, and } g(t) = \cos t \text{ is continuous everywhere. Thus } F \text{ is continuous on its domain } \{(x, y) \mid y \leq x + 1\}. \]
6. (a) The graph of $f$ decreases if we start at $(-1, 2)$ and move in the positive $x$-direction, so $f_x(-1, 2)$ is negative.

(b) The graph of $f$ decreases if we start at $(-1, 2)$ and move in the positive $y$-direction, so $f_y(-1, 2)$ is negative.
16. \( f(x, y) = x^4y^3 + 8x^2y \Rightarrow \)

\[
f_x(x, y) = 4x^3 \cdot y^3 + 8 \cdot 2x \cdot y = 4x^3y^3 + 16xy, \quad f_y(x, y) = x^4 \cdot 3y^2 + 8x^2 \cdot 1 = 3x^4y^2 + 8x^2
\]
20. \( z = \tan xy \) \quad \Rightarrow \quad \frac{\partial z}{\partial x} = (\sec^2 xy)(y) = y \sec^2 xy, \quad \frac{\partial z}{\partial y} = (\sec^2 xy)(x) = x \sec^2 xy
52. \(f(x, y) = \sin^2(mx + ny) \Rightarrow f_x(x, y) = 2 \sin(mx + ny) \cos(mx + ny) \cdot m = m \sin(2mx + 2ny)\) [using the identity \(\sin 2\theta = 2 \sin \theta \cos \theta\)], \(f_y(x, y) = 2 \sin(mx + ny) \cos(mx + ny) \cdot n = n \sin(2mx + 2ny)\).

Then \(f_{xx}(x, y) = m \cos(2mx + 2ny) \cdot 2m = 2m^2 \cos(2mx + 2ny)\),
\(f_{xy}(x, y) = m \cos(2mx + 2ny) \cdot 2n = 2mn \cos(2mx + 2ny)\),
\(f_{yx}(x, y) = n \cos(2mx + 2ny) \cdot 2m = 2mn \cos(2mx + 2ny)\), and
\(f_{yy}(x, y) = n \cos(2mx + 2ny) \cdot 2n = 2n^2 \cos(2mx + 2ny)\).