1. Find an explicit solution for the following differential equation for \( y(x) \):

\[
\frac{dy}{dx} = \pi \sqrt{xy}.
\]

**Solution:** The intent of this problem was that you recognize immediately that it is **separable**. Thus, we rewrite it in the suggestive form

\[
\frac{dy}{\sqrt{y}} = \pi \sqrt{x} \, dx.
\]

From here, we can just integrate both sides, on the left: with respect to \( y \) and on the right: with respect to \( x \), yielding

\[
\int \frac{dy}{\sqrt{y}} = \int \pi \sqrt{x} \, dx.
\]

\[
2\sqrt{y} = \frac{2\pi}{3} x^{3/2} + C,
\]

which we can solve explicitly for \( y \) to yield

\[
y = \left( \frac{\pi}{3} x^{3/2} + C \right)^2.
\]

Note, we don’t have an initial condition, so we’re left with a family of solutions described by some parameter \( C \).
2. Find an explicit solution for the following differential equation for $y(x)$:

$$xy' - 3y = x^3.$$ 

**Solution:** Unlike the first problem, this is not separable, but rather a first order, linear differential equation. Thus, we are going to use an integrating factor. Our general form of which is

$$y'(x) + P(x)y = Q(x).$$

When we rewrite our equation in this form, we see

$$y'(x) - \frac{3}{x}y = x^2 \quad \Rightarrow \quad P(x) = -\frac{3}{x}, \quad Q(x) = x^2.$$ 

Our integrating factor is then

$$\mu(x) = e^{\int P(x) \, dx} = e^{\int -3/x \, dx} = e^{-3 \ln x} = x^{-3}.$$ 

We know that for a general first order, linear ODE, after multiplying both sides of the equation our integrating factor, we’re left with

$$\frac{d}{dx} \{\mu(x)y(x)\} = \mu(x)Q(x).$$

In our particular case, that is

$$\frac{d}{dx} \{x^{-3}y(x)\} = \frac{1}{x}.$$ 

When we integrate both sides, this yields

$$x^{-3}y(x) = \ln x + C,$$

which then suggests that, solving for $y(x)$

$$y(x) = x^3 \ln x + Cx^3.$$ 

Again, due to the lack of an initial condition, we have no way of solving for $C$, but this is perfectly fine.