1. Find the equation to the tangent line to the curve at the specified point:

\[
y = \frac{2^x}{\cos x}, \quad (0, 1)
\]

**Solution:** To compute the tangent line, we need the derivative \( dy/dx \). There are two ways, either quotient rule or product rule. First, we will attempt the quotient rule method by letting \( f(x) = 2^x \) and \( g(x) = \cos x \). Then, we know:

\[
\frac{dy}{dx} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}.
\]

We know from class that:

\[
\frac{d}{dx} (a^x) = \frac{d}{dx} (e^{\ln a x}) = e^{\ln a x} \frac{d}{dx} (\ln a x) = \ln(a) a^x
\]

Thus, we can conclude \( f'(x) = \ln(2) 2^x \). We also know from class that \( g'(x) = -\sin(x) \), thus piecing this all together:

\[
\frac{dy}{dx} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} = \frac{\ln(2) 2^x \cos x - (-\sin x) 2^x}{(\cos x)^2} = \frac{2^x [\ln(2) \cos x + \sin x]}{\cos^2 x}.
\]

Evaluating at \( x = 0 \) is nontrivial, but doable. Notice we can split this sum:

\[
\frac{2^x [\ln(2) \cos x + \sin x]}{\cos^2 x} = 2^x \left[ \ln(2) \frac{1}{\cos x} + \frac{\tan x}{\cos x} \right]
\]

Now, consider \( \lim x \to 0 \), in which case \( 2^x \to 1 \), meaning it does not matter. We also know that \( \tan x \to 0 \) as \( x \to 0 \) and \( \cos x \to 1 \) as \( x \to 0 \), meaning the second term goes to 0 and the first term goes to \( \ln(2) \). Thus, we’re left with just \( \ln(2) \). Thus, \( f'(0) = \ln 2 \), meaning our tangent line is \( y - 1 = \ln(2) (x - 0) \implies y = \ln(2)x + 1 \).

Note, we could have obtained this also with product rule by rewriting \( y = 2^x \sec x \) and recalling that \( d/dx \sec = \tan x \sec x \), providing us with the same derivative.
2. Compute the derivative of each of the following functions:

(a) \( f(x) = \sqrt[3]{x^6 + 3x^5 + 4x - 2} \).

**Solution:** Recall our rule for derivatives of this form:

\[
\frac{d}{dx} \left\{ (\phi(x))^n \right\} = n(\phi(x))^{n-1} \phi'(x).
\]

Here, \( \phi(x) = x^6 + 3x^5 + 4x - 2 \) and \( n = \frac{1}{3} \). By the power rule, \( \phi'(x) = 6x^5 + 15x^4 + 4 \), meaning we can plug this into our rule:

\[
f'(x) = \frac{d}{dx} \left\{ (\phi(x))^n \right\} = n(\phi(x))^{n-1} \phi'(x) = \frac{1}{3} \left[ x^6 + 3x^5 + 4x - 2 \right]^{-2/3} (6x^5 + 15x^4 + 4) .
\]

(b) \( g(x) = e^{x \sin x} \).

**Solution:** Recall our solution to derivatives of the following form:

\[
\frac{d}{dx} \left\{ e^{u(x)} \right\} = e^{u(x)} u'(x).
\]

Here, our \( u(x) = x \sin x \), which by product rule is \( u'(x) = 1 \cdot \sin x + x \cos x \). Thus, we can evaluate our derivative using the above rule:

\[
g'(x) = \frac{d}{dx} \left\{ e^{u(x)} \right\} = e^{u(x)} u'(x) = e^{x \sin x} (\sin x + x \cos x) .
\]