Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

1. Consider the function \( g(x) \) whose graph is given by:

![Graph of \( g(x) \)](image)

State the value of each below if it exists. If it does not, state why.

(a) \( \lim_{x \to 0} g(x) \)

**Solution:** We can look at the value of \( g(x) \) as \( x \) gets really close to 0 from the left and it’s clear it approaches 3. From the right, \( g(x) \) also approaches 3. As discussed in class, the function \( g(x) \) is “nice” around \( x = 0 \), meaning you can effectively just read off the value from the graph.

(b) \( \lim_{x \to 3^-} g(x) \)

**Solution:** This notation means checking the value of \( g(x) \) as \( x \) gets close to 3 from the left. From the graph, it’s clear this value is 4.

(c) \( \lim_{x \to 3^+} g(x) \)

**Solution:** Similar to above, this notation means checking the value of \( g(x) \) as \( x \) gets close to 3 from the right. Here, that value is 2.

(d) \( \lim_{x \to 3} g(x) \)

**Solution:** Notice that the above two limits differ, and for the limit as \( x \to 3 \) to exist, we need them to be the same. Thus, the limit does not exist.

(e) \( g(3) \)

**Solution:** Note, even though the limit of the function as \( x \to 3 \) does not exist, from the graph, it’s clear \( g(3) \) does exist and corresponds to the shaded in circle which suggests \( g(3) = 3 \).
2. Compute the following limits if they exist. If they do not exist, state why.

(a) \( \lim_{x \to 3} x^2 + 3x - 5 \)

**Solution:** As we discussed in class, polynomials are “nice” and don’t cause any problems as far as limits (which you can convince yourself if you draw the picture). Thus, the limit as \( x \to 3 \) is just the value of the function at \( x = 3 \), meaning we can simply plug in \( x = 3 \) to the above function and we end up with \((3)^2 + 3 \cdot 3 - 5 = 13\).

(b) \( \lim_{x \to 0} \frac{1}{x} \)

**Solution:** Although it seems like many of you did this with a calculator, it’s easy to reason just by considering the behavior of this function as \( x \to 0 \). If \( x \) is positive, that is, approaching the limit from the right, we see that \( \frac{1}{x} \) corresponds to dividing by a small positive number and is therefore a huge positive number. Similarly, as \( x \) approaches 0 from the left, we’re dividing by a small negative number, meaning we’re left with a huge negative number. By this reasoning, it’s impossible for a huge negative number and a huge positive number to be equal, so the limit cannot exist. You can see this from the graph below: