2.8.6] I like this problem because there are several layers to it. First, which graph is f? f'?

Notice at x = 1, the blue graph has slope and the red function has a zero. Can we conclude blue' = red? NO!

Notice to the right of x = 1 blue is decreasing, but red > 0, thus blue' ≠ red.

Thus, red' = blue, but we can see this by examining the other peaks.
From this, we can see that $b = f''$, meaning $f''(c)$ occurs at blue peak and is $0$. This also corresponds to an inflection point in the red graph. $f''(c)$ is some positive number and is therefore greater.

2.8.12 Whenever $f'' > 0$, the particle is moving left (getting closer to $0$).

$f'' > 0$ moves right.

Positive acceleration $\Rightarrow f'' > 0$

Which is concave up "upside up".

i.e., from $[4, 6]$
3.1.10 \[ h(x) = (x-2)(2x+3) \]

We use the product rule:

\[
\frac{d}{dx} (h(x)) = \frac{d}{dx}(x-2) \cdot (2x+3) + (x-2) \cdot \frac{d}{dx}(2x+3)
\]

\[
= 1 \cdot 2x + 3 +
\]

Whoops. We don't use the product rule yet. That's section 3.2.

Expand: \[(x-2)(2x+3) = 2x^2 + 3x - 4x - 6\]

\[= 2x^2 - x - 6\]

Use the power rule:

\[
\frac{d}{dx} (2x^2 - x - 6) = 4x - 1
\]

Easier than product rule anyway.

3.1.28 \[ y = x^4 + 2x^2 - x \] @ (12),

\[
\frac{d}{dx} (x^4 + 2x^2 - x) = 4x^3 + 4x - 1.
\]
Thus, \( f'(1) = 7 \)
and the tangent line is:
\[ y - 2 = 7(x - 1). \]

3.1.30 \( y = (1 + 2x)^2 \) at \((1, 4)\)
\[ = 1 + 4x + 4x^2. \]

\[
\frac{d}{dx} (1 + 4x + 4x^2) = 4 + 8x,
\]

\[ f'(1) = 12 = \text{slope of tangent line}. \]

Tangent: \( y - 4 = 12(x - 1) \)
Normal line's slope: \( \frac{-1}{f'(1)} = -\frac{1}{12} \)

Normal line:
\[ y - 4 = -\frac{1}{12}(x - 1). \]
Same point, different slope.
3.1.22) \( y = ae^{-\frac{b}{v}} + cv^2 \)

\[ = ae^v + bv^{-1} + cv^2 \]

\[ \frac{d}{dv} \left( ae^v + bv^{-1} + cv^2 \right) \]

\[ = ae^v + (-1)bv^{-2} + (-2)cv^3 \]

\[ = ae^v - \frac{b}{v^2} - \frac{2c}{v^3} \]

3.1.62) I don't want to go too deeply into this problem but philosophically I think it's next. You'll see a ton of differential equations later so this is just a taste.

If we plug in the supposed solution

\[ y = Ax^2 + Bx + c \] to our DE,

\[ y'' + y' - 2y = x^2 \]
What we're left with is:

\[ y'' = 2A \]
\[ y' = 2Ax + B \]
\[ y = Ax^2 + Bx + C, \]

so

\[ y'' + y' - 2y = x^2 \implies \]

\[ 2A + 2Ax + B - 2(Ax^2 + Bx + C) = x^2, \]

The big idea here is to group powers of \( x \).

\[ (-2A)x^2 + (2A - 2B)x + 2A + B + C = x^2. \]

Notice that the coefficient of \( x^2 \) \( x \), and the constant terms must match on both sides, telling us.
\[ -2A - 1 \quad \left( x^2 \text{ coeff} \right) \]
\[ 2A - 2B = 0 \quad \left( x \text{ coeff} \right) \]
\[ 2A + B + C = 0 \quad \left( \text{constant coeff} \right) \]

\[ A = \frac{1}{2} \]
\[ B = \frac{1}{2} \]
\[ C = -\frac{1}{2} \]

This method is called "undetermined coefficients" because all we had to do was determine \( A, B, C \) to solve the differential equation.

3.2.10

\[ R(t) = (t + e^t)(3 - 5t) \]

Product rule:

\[
\frac{d}{dx} \left( R(t) \right) = \frac{d}{dx} \left( (t + e^t)(3 - 5t) \right) + \frac{d}{dx} \left( (3 - 5t)^3 \right) \cdot (t + e^t)
\]
\[ \frac{d^2}{dw} = \frac{d}{dw} (w^{3/2}) \cdot (w + ce^w) \]
\[ + \frac{d}{dw} (w + ce^w) \cdot w^{3/2} \]
\[ = \frac{3}{2} w^{1/2} (w + ce^w) \]
\[ + (1 + ce^w) w^{3/2} \]

3.2.22) \[ f'(x) = \frac{1 - xe^x}{x + e^x} \cdot g(x) \]
\[ \frac{f'(x)}{x} = \frac{g'(x) - h'(x)}{h(x)^2} \quad g' = \text{by product rule} \]
\[ -e^x - xe^x = -e^{fx} \]
\[ = -e^x \cdot (1 + x) - (1 + e^x)(1 - xe^x) \]
\[ \frac{1}{(x + e^x)^2} \]
\[ \text{Something} \]
3.2. (a) \( h = 5f - 4g \)

\[ h' = 5f' - 4g' \]

\[ h''(2) = 5f'(2) - 4g'(2) \]

\[ = 5 \cdot 2 - 4 \cdot 7 \]

\[ = -10 - 28 = -38. \]

(b) \( h(x) = f(x)g(x) \)

\[ h'(x) = f(x)g'(x) + g(x)f'(x) \]

\[ h'(2) = f'(2)g(2) + g'(2)f(2) \]

\[ = -2 \cdot 4 + 7 \cdot 3 \]

\[ = 9. \]

(c) \( v(x) = \frac{f}{g} \)

\[ h = \frac{f'g - g'f}{g^2} \]

\[ h'(2) = \frac{f'(2)g(2) - g'(2)f(2)}{g(2)^2} \]

\[ = \frac{1}{16} \]
\[ h'(x) = \frac{d}{dx} \left( \frac{1}{3.6 x^3} \cdot (1 + f(x)) \right) - \frac{d}{dx} \left( \frac{1}{3} (1 + f(x))^5 \cdot g(x) \right) \]

\[ \frac{1}{(1 + f(x))^2} \]

\[ h'(x) = g'(x) (1 + f(2)) - f'(x) g(2) \]

\[ \frac{1}{(1 + f(2))^2} \]

\[ = 7 \cdot (1 + 3) - (2)(4) \]

\[ = \frac{-14 + 8}{4} = -\frac{3}{2} \]