2.2.6) (a.) \( \lim_{x \to -3} h(x) \).

Note, even though it is a weird shape, \( h(x) \) approaches 4 as \( x \to -3 \) from the left.

(c.b.) \( \lim_{x \to -3^+} h(x) = 4 \).

Same as above, except from the right.

(c.) By (b) and (a) matching, the limit must exist and is defined to be

\( \lim_{x \to -3} h(x) = 4 \).

(d.) \( h(-3) \) is undefined. No shaded circle.

(e.) \( \lim_{x \to -7^0} h(x) = 1 \).

(f.) \( \lim_{x \to -7^1} h(x) = -1 \)

(g.) \( \lim_{x \to 50} h(x) \ DNE \) because (e), (f) differ.
(i) $h(0) = 1$ by the shaded in circle.
(ii) $\lim_{{x \to \pi}} h(x) = 2$
(iii) $h(2) = \text{ONE}.$
(iv) $\lim_{{x \to 3^{-}}} h(x) = \text{ONE. Note, this is similar to our example in class.}$
(v) $\lim_{{x \to 3^{+}}} h(x)$.
It appears as though $h(x)$ approaches 3 from the right.
Thus, $\lim_{{x \to 3^{+}}} h(x) = 3.$
Despite the mess on the left side, which we can ignore when taking the right hand limit.
2.2.10) \( f(x) = \frac{x^2 + x}{\sqrt{x^3 + x^2}}. \)

\[ \lim_{x \to 0^-} f(x). \] This problem is harder than I originally thought.

First point. \( x < 0 \) means \( x \) is negative, is that okay?

We need \( x^3 + x^2 \geq 0 \) for \( \sqrt{c} \) to make sense. Therefore:

\[
x^3 + x^2 \geq 0,
\]

\[
x^2(1 + x) \geq 0
\]

\[
\text{always } \geq 0 \quad 1 + x \geq 0 \Rightarrow x \geq -1.
\]

Thus, in the region around/near 0, we are okay.

But what is \( \lim_{x \to 0} \)? Note:

\[
\lim_{x \to 0} \frac{x^2 + x}{\sqrt{x^3 + x^2}} = \frac{x(x+1)}{\sqrt{x^2(x+1)}} = \frac{x(x+1)}{|x| \sqrt{x+1}}.
\]
\[ \frac{x}{|x|} = \text{sgn}(x). \] Remembering it gives you the sign of \( x \).

Thus we have:

\[ \lim_{x \to 0^-} \text{sgn}(x) \cdot \frac{x+1}{\sqrt{x+1}}. \]

Clearly:

\[ \lim_{x \to 0^-} \frac{x+1}{\sqrt{x+1}} = 0, \]

Thus we're left with:

\[ \lim_{x \to 0^-} \text{sgn}(x) = -1. \]

Similarly, \( \lim_{x \to 0^+} f(x) \) boils down to:

\[ \lim_{x \to 0^+} \text{sgn}(x) = 1. \]

Thus \( \lim_{x \to 0} f(x) \) DNE!\]
2.2.16) We want:

\[ \lim_{x \to -70^-} f(x) = 2 \quad \lim_{x \to 70^+} f(x) = 0 \quad \lim_{x \to -4} f(x) = 3 \]

\[ \lim_{x \to 74^+} f(x) = 0, \quad f(0) = 2, \quad f(-4) = 1. \]

One such example:

2.3.10) \[ \lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}. \] It's a rational expression, what can we do? Plug 4 in! \[ \checkmark \]
\[
\frac{(v)^2 - 4(v)}{(v)^2 - 3(v) - 4} = \frac{0}{16 - 12 - 4} = \frac{0}{0} \Rightarrow \\
This is bad, we have to try something else!
\]

\[
\lim_{x \to 4} \frac{x(x-4)}{(x-4)(x+1)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}.
\]

because \( x \neq 4 \).

2.3.16) \( \lim_{h \to 0} \frac{(2+h)^3 - 8}{h} \)  
If we plug in \( h = 0 \), divide by 0, which is bad. What do we do?

Expand:
\[
(2+h)^3 - 8 = 8 + h^3 + 6h^2 + 12h - 8 \\
= h(h^2 + 6h + 12).
\]
Thus:
\[
\lim_{h \to 0} \frac{(2+h)^3 - 8}{h} = \lim_{h \to 0} \frac{h(2h^2 + 6h + 12)}{h} = \lim_{h \to 0} (h^2 + 6h + 12) = 12.
\]

23.24)
\[
\lim_{x \to 7-4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}.
\]

Our standard trick: multiply by conjugate:
\[
\lim_{x \to 7-4} \frac{\sqrt{x^2 + 9} - 5}{x + 4} \cdot \frac{\sqrt{x^2 + 9} + 5}{\sqrt{x^2 + 9} + 5} = \lim_{x \to 7-4} \frac{x^2 + 25}{(x + 4)(\sqrt{x^2 + 9} + 5)} = \lim_{x \to 7-4} \frac{x^2 + 25}{x^2 - 16} = \lim_{x \to 7-4} \frac{(x+4)(x-4)}{(x+4)(\sqrt{x^2 + 9} + 5)} = \lim_{x \to 7-4} \frac{x-4}{\sqrt{x^2 + 9} + 5} = \frac{3}{10} = \frac{3}{10}.
\]
2.3.2) We want to prove that
\[ \lim_{x \to 0^+} x \cdot \sin(\pi/x) = 0. \]
This should scream **Squeeze theorem**.

\[ \text{meaning we want } f(x), g(x). \]

\[ f(x) \leq x \cdot \sin(\pi/x) \leq g(x) \]

where \[ \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} g(x) = 0 \]

Typically, we start by noticing that \(\sin\) or \(\cos\) are bounded:

\[ -1 \leq \sin \leq 1. \]

but we have
\[ e^{\sin x}, \text{ resulting in:} \]

\[ e^{-1} \leq e^{\sin x} \leq e^{1}. \]

But we want \( \int e^{\sin x} \), so multiply by \( \int e^{1} \):

\[ \frac{\int x}{e} \leq \int x e^{\sin x} \leq e^{\int x}. \]

Notice, \( \lim_{x \to 0^+} g(x) = 0 \)

\( \lim_{x \to 0^+} h(x) = 0 \).

Thus, the limit of \( f \) is squeezed between these two, yielding:

\[ \lim_{x \to 0^+} f(x) = 0 \quad \text{as desired} \]