**Section 3.1: The Rectangular Coordinate System**

<table>
<thead>
<tr>
<th>Rectangular coordinate system or Cartesian plane</th>
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<tbody>
<tr>
<td>The points in the coordinate plane are described by two coordinates: the $x$-coordinate gives the point’s horizontal position relative to the origin, the $y$-coordinate gives the point’s vertical position relative to the origin.</td>
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<td>We use the $x$-axis for the independent variable and the $y$-axis for the dependent variable. The axes divide the coordinate plane in 4 quadrants.</td>
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Plotting a point
Locating a point in the Cartesian plane is called *plotting* a point.

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**Ex.1 Plotting points.**
Plot the following points in the Cartesian plane.

1. (1, 3)
2. (−1, 2)
3. (0, 0)
4. (2, −1)
5. (−2, −3)

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**Ex.2 Finding coordinates of points.**
Determine the coordinates of each of the points shown below.
Ex.3 The population (in millions) in California from 1990 to 2000 is shown in the following table.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>30.0</td>
<td>30.5</td>
<td>31.0</td>
<td>31.3</td>
<td>31.5</td>
<td>31.7</td>
<td>32.0</td>
<td>32.5</td>
<td>33.0</td>
<td>33.5</td>
<td>34.0</td>
</tr>
</tbody>
</table>

Sketch a scatter plot of the data.

Problem solving
There are 3 different approaches to solve a problem.

1. **Algebraic approach**: use algebra to find the solution.
2. **Numerical approach**: construct a table.
3. **Graphical approach**: draw a graph.

Ex.4
Construct a table of values for \( y = 3x + 2 \). Then plot the solution points on a rectangular coordinate system. Choose \( x \)-values of \(-2, -1, 0, 1, \text{ and } 2\).
Verifying solutions
To verify that \((x, y)\) is a solution of an equation with variables \(x\) and \(y\), follow the steps:
1. Substitute the values of \(x\) and \(y\) into the equation.
2. Simplify each side of the equation.
3. If each side simplifies to the same number, then \((x, y)\) is a solution of the equation. Otherwise, it is not a solution.

Ex.5 Verifying solutions of an equation.
Which of the ordered pairs: \((2, 1)\), \((0, -3)\), \((-2, -5)\), and \((1, -\frac{5}{2})\) are solutions of \(x^2 - 2y = 6\)?

Ex.6
(1) Find the vertical distance between the points \((2, -2)\) and \((2, 4)\).
(2) Find the horizontal distance between the points \((3, -2)\) and \((2, -2)\).
The distance formula

The distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex. 7

Find the distance between the points $(-1, 2)$ and $(2, 4)$. 
Ex. 8
Show that the points (1, 2), (3, 1), and (4, 3) are vertices of a right triangle.

Definition of collinear points
Three or more points are collinear if they lie on the same line.

Ex. 9
Determine whether the set of points \{A = (2, 6), B = (5, 2), C = (8, −2)\} is collinear.

The midpoint formula
The midpoint of a line segment that joins two points \((x_1, y_1)\) and \((x_2, y_2)\) is the point that divides the segment into two equal parts and it is given by

\[
\text{midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
Ex. 10
Find the midpoint of the line segment joining the points \((-5, -3)\) and \((9, 3)\).

Section 3.2: Graphs of Equations

Definition of graph
The set of all solution points of an equation is called its graph.

The point-plotting method of sketching a graph
In order to graph an equation, you need to follow these steps:
1. If possible, rewrite the equation by isolating one of the variables.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth line or curve.

Ex. 1
Sketch the graph of \(3x - y = 2\).
Ex. 2
Sketch the graph of $-x^2 + 2x + y = 0$.

Ex. 3
Sketch the graph of $y = |x - 2|$.
### Definition of intercepts

- The point \((a, 0)\) is called an *x-intercept* of the graph of an equation if it is a solution point of the equation. To find the \(x\)-intercepts, let \(y = 0\) and solve the equation for \(x\).
- The point \((0, b)\) is called a *y-intercept* of the graph of an equation if it is a solution point of the equation. To find the \(y\)-intercepts, let \(x = 0\) and solve the equation for \(y\).

### Ex. 4

Find the intercepts and sketch the graph of \(y = 2x - 3\).
Section 3.3: Slope and Graphs of Linear Equations

Definition of the slope of a line
The slope $m$ of the nonvertical line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.
A line with positive slope ($m > 0$) rises from left to right.
A line with negative slope ($m < 0$) falls from left to right.
A line with zero slope ($m = 0$) is horizontal. The equation for such a line is $y = b$.
A line with undefined slope is vertical. The equation for such a line is $x = a$.

Ex. 1
Find the slope of the line passing through each pair of points and describe each line.

(1) (1, 2) and (4, 5)
(2) (−1, 4) and (2, 1)
(3) (1, 4) and (3, 4)
(4) (3, 1) and (3, 3)
Ex.2
Sketch the graph of the line given by $2x + 3y = 6$.

Slope-intercept form of a line
The graph of the equation $y = mx + b$ is a line whose slope is $m$ and whose $y$-intercept is $(0, b)$.

Ex.3
Find the slope and $y$-intercepts of the graph of the equation $4x - 5y = 15$. Sketch the graph of the line.
Ex.4
Use the slope and $y$-intercept to sketch the graph of $12x + 3y = 6$.

Parallel and perpendicular lines
Two distinct nonvertical lines are parallel if they have the same slope. Consider two nonvertical lines whose slopes are $m_1$ and $m_2$. Two lines are perpendicular if and only if their slopes are negative reciprocals. That is

$$m_1 = -\frac{1}{m_2} \text{ or } m_1 \cdot m_2 = -1$$

Ex.5
Are the pairs of lines parallel, perpendicular, or neither?
1. $y = -2x + 4$, $y = \frac{1}{2}x + 1$
2. $y = \frac{1}{3}x + 2$, $y = \frac{1}{3}x - 3$
Section 3.4: Equations of Lines

Point-slope form of the equation of a line

The point-slope form of the equation of the line that passes through the point \((x_1, y_1)\) and has a slope of \(m\) is

\[
y - y_1 = m(x - x_1)
\]

Ex. 1

Write an equation of the line that passes through the point \((2, -3)\) and has slope \(m = -2\).
**Point-slope form of the equation of a line**
The general form of the equation of the line is
\[ ax + by + c = 0 \]

**Two-point-slope form of the equation of a line**
The two-point form of the equation of the line that passes through the points \((x_1, y_1)\) and \((x_2, y_2)\) is
\[ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \]

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**Ex.2**
Write the general form of the equation of the line that passes through the points \((-2, 3)\) and \((4, 2)\).
Ex.3 Write equations of horizontal and vertical lines. Write an equation for each line.
(1) Vertical line through \((-2, 4)\).
(2) Horizontal line through \((0, 6)\).
(3) Line passing through \((-2, 3)\) and \((-1, 3)\).
(4) Line passing through \((-1, 2)\) and \((-1, 3)\).

Ex.4 Write equations of the lines that pass through the point \((3, -2)\) and are
(1) parallel
(2) perpendicular
to the line \(x - 4y = 6\).
Linear inequalities
A linear inequality in two variables, $x$ and $y$, is an inequality that can be written in one of the following forms:

$$ax + by < c, \quad ax + by > c, \quad ax + by \leq c, \quad ax + by \geq c$$

where $a$ and $b$ are not both zero.

Ex.1 Verifying solutions of linear inequalities.
Determine whether each point is a solution of $2x - 3y \geq -2$.

1. $(0, 0)$
2. $(0, 1)$

Graph of a linear inequality in two variables
The graph of a linear inequality is the collection of all solution points of the inequality. To sketch the graph of a linear inequality you need to do the following steps:

1. Replace the inequality symbol with an equal sign and sketch the graph of the equation. Use a dashed line for $<$ or $>$, and a solid line for $\leq$ or $\geq$.
2. The graph of the equation separates the plane into two regions, called half-planes. Test one point in one of the half-planes formed by the graph in Step 1.
   a. If the point satisfies the inequality, shade the entire half-plane to denote that every point in the region satisfies the inequality.
   b. If the point does not satisfy the inequality, then shade the other half-plane.
Ex.2
Sketch the graph of each inequality.
(1) $x \geq -3$
(2) $y < 4$

Ex.3
Sketch the graph of the inequality $x + y > 3$. 
Ex. 4
Sketch the graph of the inequality $2x + y \leq 2$.

Ex. 5
Use the slope of a linear equation to sketch the graph of the inequality $2x - 3y \leq 15$. 
Section 3.6: Relations and Functions

**Definition of a function**

A function $f$ from a set $A$ to a set $B$ is a rule of correspondence that assigns to each element $x$ in the set $A$ exactly one element $y$ in the set $B$. The set $A$ is called the domain of the function $f$. The range of the function is the set of elements in $B$ that are in correspondence with elements in $A$.

Functions are represented in four ways:

1. Verbally by a sentence that describes how the input variable is related to the output variable.
2. Numerically by a table or a list of ordered pairs that matches input values with output values.
3. Graphically by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis.
4. Algebraically by an equation in two variables.

In the case of functions described as equations, the variable $x$ is the independent variable and the variable $y$ is the dependent variable.

In order to understand if you have a function, you must keep in mind that

1. each element in the domain $A$ must be matched with an element in the range contained in $B$.
2. some elements in the set $B$ may not be matched with any element in the domain $A$.
3. two or more elements in the domain may be matched with the same element in the range.
4. no elements of the domain is matched with two different elements in the range.

**Ex.1**

Decide whether or not the description represents a function.

1. $\{(1, 1), (1, 3), (2, 5)\}$
2. $\{(1, 2), (2, 3), (3, 2)\}$
3. $y = x^2 + 1$
4. $-2x + 3y = 4$
Function notation
In general a function is denoted as \( f(x) \) (read \( f \) of \( x \)), where \( f \) is the name of the function, \( x \) is the domain value and \( f(x) \) is the range value \( y \) for a given \( x \). The process of finding the value of \( f(x) \) for a given value of \( x \) is called evaluating a function.

Ex.2
Let \( g(x) = 3x - 4 \). Find each value of the function.
(1) \( g(1) \)
(2) \( g(-2) \)
(3) \( g(y) \)
(4) \( g(x + 1) \)
(5) \( g(x) + g(1) \)
Ex.3

Let

\[ f(x) = \begin{cases} 
    x^2 + 1, & \text{if } x < 0 \\
    x - 2, & \text{if } x \geq 0 
\end{cases} \]

Find each value of the function.

1. \( f(-1) \)
2. \( f(0) \)
3. \( f(-2) \)
4. \( f(-3) + f(4) \)
Ex.4
Find the domain of each function.
(1) \( f(x) = \frac{1}{x-3} \)
(2) \( f(x) = \sqrt{2x - 6} \)
(3) \( f(x) = \frac{4x}{(x-1)(x+5)} \)