MATH 3160: APPLIED COMPLEX VARIABLES
FINAL EXAM (VERSION A)

Name:

This test has 7 pages and 6 problems.
No calculators are allowed. You are permitted only a pencil or pen. Work out everything as far as you can before making decimal approximations.

1. Calculate

$$\operatorname{Arg} \frac{1}{1+i}$$

Date: May 2, 2001.

2. Is $u(x, y) = e^{-y} \cos x$ the real part of an analytic function? If so, find the imaginary part of that function.

3. **(a)** Give two nonzero terms in the Laurent expansion of

$$\cos(1/z)\sin(1/z)$$

about $z = 0$.

(b) What sort of singular point does this function have at $z = 0$?

(c) Find

$$\int_{|z|=1} \cos(1/z)\sin(1/z)dz\ .$$

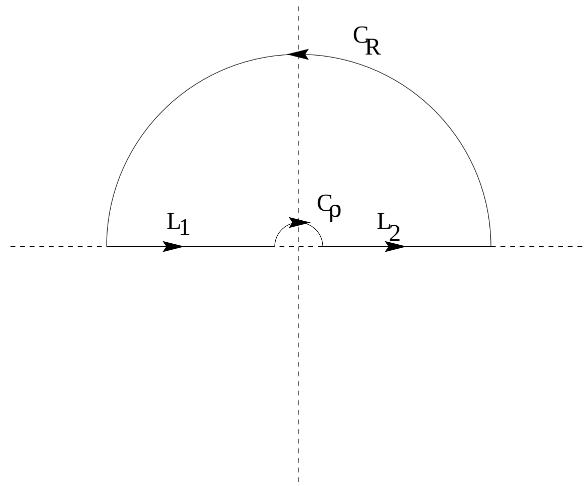

FIGURE 1. A contour, with ρ (the radius of C_ρ) small, and R (the radius of C_R) large.

4. Show that
$$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \pi$$
by integration of the function
$$f(z) = \frac{e^{iz}}{z}$$
along a contour like that shown in figure 1.

5. Using the principal branch of the square root, let

$$Z = \sqrt{\sin z}\ .$$

Where does it map the half strip

$$0 < x < \pi/2,\ y > 0?$$

Draw the boundaries of this half strip, and show where they go under this map.

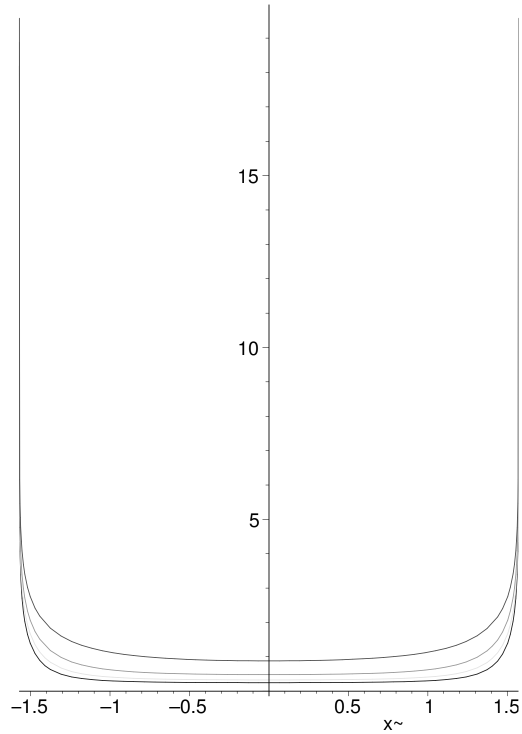

FIGURE 2. The stream lines of the flow whose complex potential is $F(z) = \sin z$

6. **(a)** Show that $F(z) = \sin z$ is the complex potential of a flow that never leaves the half strip

$$-\pi/2 \leq x \leq \pi/2,\ y \geq 0 .$$

(b) Calculate the equations for all of the stream lines. Write them in the form of y as a function of x. (They should look like the ones drawn in figure 2.)

(c) Show that the stream function is positive everywhere in the half strip.

(d) Show from the definition of a stream line (level curves of the stream function) that the only critical point of y as a function of x on each stream line is at $x = 0$. Hint: implicitly differentiate the equation $\psi = c_0$, where ψ is the stream function.

(e) Show that the closest any stream line gets to the x axis is at the point

$$x = 0,\ y = \ln\left(c_0 + \sqrt{c_0^2 + 1}\right) .$$

(f) Show that the stream line passing through the point

$$x = \frac{\pi}{4}, \ y = \ln\sqrt{2}$$

never goes closer to the x axis than a distance

$$\ln\left(\frac{1+\sqrt{17}}{4}\right).$$