A.5 - Solving Equations

↳ In this section, we work on solving equations. That means figuring out what value(s) for the variable makes the equation true.

↳ One method we use, is doing the same thing to both sides.

Ex: A linear equation

$2(3x-5) = 8$ → Distribute

$6x - 10 = 8$ → $+10$

$6x = 18$ → $\div 6$

$x = 3$

check: $2(3\cdot 3-5) = 2(9-5) = 2(4) = 8$ ✓

Ex: Fractions

$$\frac{2x}{5} + \frac{3x}{2} = 2$$

2 methods

1) ↳ multiply both sides of eqn by common denominator to remove fractions

2) ↳ or add w/ common denominator

1) $\left(\frac{2x}{5} + \frac{3x}{2} = 2\right)10 \Rightarrow 4x + 15x = 20 \Rightarrow 19x = 20 \Rightarrow x = 20/19$

2) $\frac{2x}{5} + \frac{3x}{2} = 2 \Rightarrow \frac{4x}{10} + \frac{15x}{10} = 2 \Rightarrow \frac{19x}{10} = 2 \Rightarrow 19x = 20 \Rightarrow x = 20/19$

→ Check the solution!

Ex More fractions:

$$\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x^2+3x} \Rightarrow \frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)}$$

↳ use method 1. Overall common denominator is $x(x+3)$

$$\left(\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)}\right) x(x+3)$$

$$\Rightarrow 6(x+3) - 2x = 3(x+5)$$

$$\Rightarrow 6x+18-2x = 3x+15$$

$$\Rightarrow 4x+18 = 3x+15$$

$$\Rightarrow x=-3$$

↳ check: $x=-3$ makes us divide by zero. It is an extraneous solution. Thus there is $\boxed{\text{no Solution}}$

↳ one type of equation we often solve is called a quadratic equation. It has the form $ax^2+bx+c=0$

↳ we have 4 basic methods for solving: factoring, extracting square roots, completing the square, and using the quadratic formula

1) factoring: Solve $x^2+10x+24=0$

$$\Rightarrow (x+4)(x+6)=0$$

so $x+4=0 \Rightarrow \boxed{x=-4}$

or $x+6=0 \Rightarrow \boxed{x=-6}$

2 solutions to a quadratic equation

Ex: Solve $2x^2-5x-3=0$

$\Rightarrow (2x+1)(x-3)=0$

so $2x+1=0 \Rightarrow \boxed{x=-1/2}$

or $x-3=0 \Rightarrow \boxed{x=3}$

2) Extract square roots

2 solutions!

<u>EX</u> Solve

a) $x^2-3=0 \Rightarrow x^2=3 \Rightarrow \boxed{x=\pm\sqrt{3}}$

b) $x^2+2x+1=5 \Rightarrow (x+1)^2=5 \Rightarrow x+1=\pm\sqrt{5} \Rightarrow \boxed{x=-1\pm\sqrt{5}}$

3) Complete the square: recall $(x+r)^2 = x^2+2rx+r^2$

<u>EX</u> a) $x^2-2x-1=0$ → move constant term

$\Rightarrow x^2-2x=1$ → divide coeff of x by 2, square it, and add it to both sides

$\Rightarrow x^2-2x+\left(\frac{-2}{2}\right)^2 = 1+\left(\frac{-2}{2}\right)^2$ ⇒ factor perfect square

$\Rightarrow (x-1)^2=2$

$\Rightarrow x-1=\pm\sqrt{2}$

$\Rightarrow \boxed{x=1\pm\sqrt{2}}$

<u>EX</u> leading coeff not 1. → Start by dividing out leading coeff.

$3x^2-2x-5=0$

$\Rightarrow x^2-\frac{2}{3}x-\frac{5}{3}=0$

$\Rightarrow x^2-\frac{2}{3}x=\frac{5}{3}$

$$\Rightarrow \quad x^2 - \frac{2}{3}x + \left(\frac{-1}{3}\right)^2 = \frac{5}{3} + \left(\frac{-1}{3}\right)^2$$

$$\Rightarrow \quad \left(x - \frac{1}{3}\right)^2 = \frac{5}{3} + \frac{1}{9}$$

$$\Rightarrow \quad \left(x - \frac{1}{3}\right)^2 = \frac{16}{9}$$

$$\Rightarrow \quad \left(x - \frac{1}{3}\right) = \pm\sqrt{\frac{16}{9}}$$

$$\Rightarrow \quad x - \frac{1}{3} = \pm\frac{4}{3}$$

$$\Rightarrow \quad \boxed{x = \frac{1}{3} \pm \frac{4}{3}}$$

4) Quadratic Formula : comes from completing the square on

$$ax^2 + bx + c = 0.$$

says $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex: $3x^2 + 4x = 1$ → standard form

$$3x^2 + 4x - 1 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)} = \frac{-4 \pm \sqrt{16 + 12}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{28}}{6}$$

$$= \frac{-4 \pm \sqrt{4 \cdot 7}}{6}$$

$$= \frac{-4 \pm 2\sqrt{7}}{6}$$

$$= \frac{2(-2 \pm \sqrt{7})}{6}$$

→ factor, then cancel!

$$\boxed{x = \frac{-2 \pm \sqrt{7}}{3}}$$

→ Completing the square and using the quadratic formula both work to solve <u>any</u> quadratic equation. Factoring only works when you're lucky.

→ Completing the square is great when the leading coefficient is 1. Using the quadratic formula may be easier when the leading coeff. is not 1.

<u>Higher Degree Polynomials</u>: Try to factor

<u>Ex:</u> $x^3 - 3x^2 + 2x = 0$

$\Rightarrow x(x^2 - 3x + 2) = 0$

$\Rightarrow x(x-2)(x-1) = 0$

$\Rightarrow x = 0, 2, 1$

<u>Equations with Radicals</u>: Isolate radical & square both sides

Ex: $-3 = \sqrt{30-2x} - x$

$\Rightarrow x - 3 = \sqrt{30-2x}$

$\Rightarrow (x-3)^2 = \left(\sqrt{30-2x}\right)^2$

square the whole side, not just the terms. If you haven't foiled, you're not doing it right!

$\Rightarrow x^2 - 6x + 9 = 30 - 2x$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x = 7, \; x = -3$

Always check! $\sqrt{30-2(7)} - 7 = \sqrt{30-14} - 7 = \sqrt{16} - 7 = 4 - 7 = -3$ ✓ ok

$\sqrt{30-2(-3)} - (-3) = \sqrt{30+6} + 3 = \sqrt{36} + 3 = 6 + 3 = 9 \neq -3$ No!

$\boxed{x = 7}$ is the only true solution

Ex 2 radicals: Isolate 1 radical & square both sides
Then isolate the other radical & square both sides

$\sqrt{2x-5} - \sqrt{x-3} = 1$

$\Rightarrow \sqrt{2x-5} = 1 + \sqrt{x-3}$

$\Rightarrow \left(\sqrt{2x-5}\right)^2 = \left(1+\sqrt{x-3}\right)^2$ FOIL!

$\Rightarrow 2x-5 = 1 + 2\sqrt{x-3} + x-3$

$\Rightarrow 2x-5 = 2\sqrt{x-3} + x-2$

$\Rightarrow x-3 = 2\sqrt{x-3}$

$\Rightarrow (x-3)^2 = \left(2\sqrt{x-3}\right)^2$

$\Rightarrow x^2-6x+9 = 4(x-3)$

$\Rightarrow x^2-6x+9 = 4x-12$

$\Rightarrow x^2-10x+21=0$

$\Rightarrow (x-3)(x-7)=0$

$\Rightarrow \boxed{x=3,\ x=7}$

Check! Both solutions work here.

Equations with Absolute Values

↳ Something like $|x|=3$ has 2 solutions. $x=3$ and $x=-3$

↳ This idea holds true for more complicated equations

Ex Solve $|x-10| = x^2-10x$

2 equations: $x-10 = x^2-10x$ and $-(x-10) = x^2-10x$

1st: $x-10=x^2-10x$

$\Rightarrow x^2-11x+10=0$

$\Rightarrow (x-1)(x-10)=0$

$x=1, 10$

check in original equation

$|1-10| \stackrel{?}{=} 1-10$

$|-9| \stackrel{?}{=} -9$

$9 \neq -9$ no!

$|10-10| \stackrel{?}{=} 10^2-(10)(10)$

$|0| \stackrel{?}{=} 100-100$

yes!

$\boxed{x=10}$

2nd: $-(x-10)=x^2-10x$

$\Rightarrow -x+10=x^2-10x$

$\Rightarrow x^2-9x-10=0$

$\Rightarrow (x-10)(x+1)=0$

$\Rightarrow x=10, -1$

Check: $|-1-10| \stackrel{?}{=} (-1)^2-10(-1)$

$|-11| \stackrel{?}{=} 1+10$

$|-11| \stackrel{?}{=} 11$

yes! $\boxed{x=-1}$

2 solutions