4 - Rational Expressions

ration expression is the quotient of 2 algebraic expressions.

↳ The domain of an expression is the set of all values where the expression is defined.

↳ for rational expressions, we don't want to divide by zero.

↳ for radicals we can't take the square root (4th root, 6th root, etc.) of a negative number.

Ex Find domain

a) $\frac{2}{x+3}$ → don't want to divide by zero so domain is all numbers except $x=-3$

b) $\frac{x-1}{x^2-x} = \frac{x-1}{(x-3)(x+2)}$ domain is all $\mathbb{R}$ except $x=3, x=-2$

c) $\sqrt{2x-5}$ → can't take square root of negative number, so need

$2x-5 \geq 0 \Rightarrow 2x \geq 5 \Rightarrow \boxed{x \geq 5/2}$ → This is the domain

Simplifying Rational Expressions : Factor & Cancel

Ex $\frac{x^2+5x+6}{x^2+6x+8} = \frac{(x+2)(x+3)}{(x+2)(x+4)} = \frac{x+3}{x+4}$; $x \neq -2$

↳ Things are only equal if they are equal for every x value. Here, the start and the end are not equal if $x=-2$ since the start is undefined and the end is $\frac{1}{2}$. So we restrict the domain at the end to match the domain at the beginning.

Ex $\dfrac{x^2+x-12}{4x^2+14x-8} = \dfrac{(x+4)(x-3)}{2(2x^2+7x-4)}$

Factors of 2	Factors of -4
1, 2	1, -4
	-1, 4
	2, -2

(2)

Guess & check all combinations until we find one that works.

$(x+1)(2x-4)$ nope
$(x-1)(2x+4)$ nope
$(x+2)(2x-2)$ nope
$(2x+1)(x-4)$ nope
$(2x-1)(x+4)$ yes!

$= \dfrac{(x+4)(x-3)}{2(2x-1)(x+4)}$

$= \dfrac{x-3}{2(2x-1)}; \; x \neq -4$

Multiply/Divide Rational Expressions: Factor & Cancel (again)

Ex $\dfrac{x^2+x-12}{4x^2+14x-8} \cdot \dfrac{2x^2-5x+2}{x^2-2x-3} = \dfrac{(x+4)(x-3)}{2(2x-1)(x+4)} \cdot \dfrac{(2x-1)(x-2)}{(x-3)(x+1)}$

$(2x-2)(x-1) \rightarrow$ nope
$(2x-1)(x-2) \rightarrow$ yes!

$= \dfrac{x-2}{2(x+1)}; \; x \neq 1/2, -4, 3$

Ex $\dfrac{x^2-2x-3}{x^2-4x+3} \div \dfrac{x-4}{2x^2+8x-10} = \dfrac{(x-3)(x+1)}{(x-3)(x-1)} \div \dfrac{x-4}{2(x^2+4x-5)}$

$= \dfrac{(x-3)(x+1)}{(x-3)(x-1)} \div \dfrac{x-4}{2(x-1)(x+5)}$

$= \dfrac{(x-3)(x+1)}{(x-3)(x-1)} \cdot \dfrac{2(x-1)(x+5)}{x-4}$

$= \dfrac{2(x+1)(x+5)}{x-4}; \; x \neq 1, 3$

Add/Subtract Rational Expressions : Need Common Denominator

Ex: $\frac{3}{x+2} + \frac{2}{2x-1}$ CD: $(x+2)(2x-1)$

$$= \frac{3(2x-1)}{(x+2)(2x-1)} + \frac{2(x+2)}{(x+2)(2x-1)}$$

$$= \frac{3(2x-1)+2(x+2)}{(x+2)(2x-1)} = \frac{6x-3+2x+4}{(x+2)(2x-1)} = \frac{8x+1}{(x+2)(2x-1)}$$

Ex: $\frac{3}{x-1} - \frac{2}{x} + \frac{x+3}{x^2-1} = \frac{3}{x-1} - \frac{2}{x} + \frac{x+3}{(x-1)(x+1)}$ CD: $x(x-1)(x+1)$

$$= \frac{3x(x+1)}{x(x-1)(x+1)} - \frac{2(x-1)(x+1)}{x(x-1)(x+1)} + \frac{x(x+3)}{x(x-1)(x+1)}$$

$$= \frac{3x^2+3x-2(x^2-1)+x^2+3x}{x(x-1)(x+1)}$$

$$= \frac{3x^2+3x-2x^2+2+x^2+3x}{x(x-1)(x+1)}$$

$$= \frac{2x^2+6x+2}{x(x-1)(x+1)}$$

$$= \frac{2(x^2+3x+1)}{x(x-1)(x+1)}$$

Simplifying Complex Fractions : work in numerator then denominator & divide

ex $\dfrac{\frac{2}{x-1}+3}{\frac{4}{x}+\frac{1}{x+1}} = \dfrac{\frac{2}{x-1}+\frac{3(x-1)}{x-1}}{\frac{4(x+1)}{x(x+1)}+\frac{1(x)}{x(x+1)}} = \dfrac{\frac{2+3x-3}{x-1}}{\frac{4x+4+x}{x(x+1)}}$

$= \dfrac{\frac{3x-1}{x-1}}{\frac{5x+4}{x(x+1)}}$ } Domain $x \neq 1, -4/5, 0, -1$

$= \dfrac{3x-1}{x-1} \cdot \dfrac{x(x+1)}{5x+4}$ } Domain $x \neq 1, -4/5$

$= \dfrac{(3x-1)(x)(x+1)}{(x-1)(5x+4)}$; $x \neq 0, -1$

Other method: Use overall common Denominator

ex $\dfrac{\frac{2}{x-1}+3}{\frac{4}{x}+\frac{1}{x+1}} = \dfrac{\frac{2}{x-1}+3}{\frac{4}{x}+\frac{1}{x+1}} \cdot \dfrac{\frac{x(x-1)(x+1)}{1}}{\frac{x(x-1)(x+1)}{1}} = \dfrac{2x(x+1)+3x(x-1)(x+1)}{4(x-1)(x+1)+x(x-1)}$

$= \dfrac{x(x+1)\left(2+3(x-1)\right)}{(x-1)\left(4(x+1)+x\right)}$

$= \dfrac{x(x+1)(3x-1)}{(x-1)(5x+4)}$; $x \neq 0, -1$

Common Errors

1) $\frac{x+2}{2} \neq \frac{x+1}{1}$ → can only cancel multiplied terms

↳ $\frac{2x+2}{2} = \frac{2(x+1)}{2} = x+1$ → the 2 is multiplied, not added

2) $\frac{x^2+2x+1}{x^2} \neq \frac{x+2+1}{x}$ → x is not in each term in the numerator

↳ Basically, if you can't factor the thing out of the numerator/~~denominator~~ denominator, you can't cancel it!