

A.3 → Polynomials and Factoring

↳ A polynomial is a common type of algebraic expression.

EX: $2x^3 + 3x^2 + 2$ is a polynomial
 ↓ ↓ ↓
 coefficients

↳ powers can't be negative and must be integers
 ↳ standard form if powers are in descending order.

Terminology:

Coefficients: numbers (with signs) multiplying the variables

Degree: ~~highest~~ highest power on variable(s)

Leading coefficient: coefficient corresponding to highest powered variable

constant term: coefficient with no variable

EX $2 + 2x - x^3 + 3x^4 - x^2$

standard form: $3x^4 - x^3 - x^2 + 2x + 2$

Degree: 4

coefficient of x^2 : -1

constant term: 2

coefficient of x : 2

leading coefficient: 3

Adding/Subtracting Polynomials : combine like terms

EX: a) $(2x^3 - 6x + 1) + (x^2 - 3x - 2) = 2x^3 + x^2 - 9x + 1$

b) $(4x^3 - 3x^2 + 12x - 5) - (3x^2 - 2x + 1)$

$= 4x^3 - 3x^2 + 12x - 5 - 3x^2 + 2x - 1$

$= 4x^3 - 6x^2 + 14x - 6$

Multiplying Polynomials : Distributive Property

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~~x~~ a) $(2x+3)(3x-1) = 6x^2 - 2x + 9x - 3 = 6x^2 + 7x - 3$

First Outer Inner Last

$$(3x-1)(x^3+2x-1) = 3x^4 + 6x^2 - 3x - x^3 - 2x + 1$$
$$= 3x^4 - x^3 + 6x^2 - 5x + 1$$

Factoring:

↳ factoring means writing as a product of polynomials

Ex: Factor out common terms

a) $3x^2 - 6x = 3x(x-2)$

b) $-3x^2 - 18x + 36 = -3(x^2 + 6x - 12)$

c) $(x^2+1)(2x) - (x^2+1)(x^2) = (x^2+1)(2x - x^2)$
 $= (x^2+1)(x)(2-x)$

Special Forms

Difference of two squares: $u^2 - v^2 = (u-v)(u+v)$

Perfect Squares: $u^2 + 2uv + v^2 = (u+v)^2$

$$u^2 - 2uv + v^2 = (u-v)^2$$

} know these!

Sum/difference of cubes: $u^3 + v^3 = (u+v)(u^2 - uv + v^2)$

$$u^3 - v^3 = (u-v)(u^2 + uv + v^2)$$

x Factor

$$a) 3 - 12x^2 = 3(1 - 4x^2) = 3(1^2 - (2x)^2) = 3(1 - 2x)(1 + 2x)$$

$$b) 2x^2 + 12x + 18 = 2(x^2 + 6x + 9) = 2(x + 3)^2$$

Factoring Trinomials: A trinomial $ax^2 + bx + c$ factors as

$$ax^2 + bx + c = \left(\frac{\quad}{\quad} x + \frac{\quad}{\quad} \right) \left(\frac{\quad}{\quad} x + \frac{\quad}{\quad} \right)$$

$\xrightarrow{\text{multiply to } c}$
 $\xrightarrow{\text{multiply to } a}$

Leading Coefficient 1:

a) $x^2 - x - 6 = (x \quad)(x \quad) \rightarrow$ find 2 numbers that multiply to -6 and add to -1 .

$$x^2 - x - 6 = (x - 3)(x + 2)$$

$\hookrightarrow -3$ and 2

b) $x^2 - 8x + 15$

\hookrightarrow multiply to 15 and add to -8 , $\rightarrow -5$ and -3

$$\therefore x^2 - 8x + 15 = (x - 5)(x - 3)$$

c) $x^2 + 12x + 32 = (x + 4)(x + 8)$

Leading Coefficient Not 1: Guess and check

x $3x^2 - x - 2 = (\quad)(\quad)$

\hookrightarrow You know the product of the first terms must be 3 and the product of the last terms must be -2 .

\hookrightarrow The only factors of 3 are 1 and 3
 factors of -2 are -1 and 2 or 1 and -2

e guess and check until we find the right combination that works. (4)

$$(3x+1)(x-2) = 3x^2 - 6x + x - 2 \rightarrow \text{nope}$$

$$(3x-1)(x+2) = 3x^2 + 6x - x - 2 \rightarrow \text{nope}$$

$$(3x-2)(x+1) = 3x^2 + 3x - 2x - 2 \rightarrow \text{nope}$$

$$(3x+2)(x-1) = 3x^2 - 3x + 2x - 2 \rightarrow \text{yep!}$$

$$3x^2 - x - 2 = (3x+2)(x-1)$$

\times Factor $6x^2 - x - 15$

Factors of 6

Factors of -15

1, 6
2, 3

1, -15
-1, 15
3, -5
-3, 5

\hookrightarrow Keep guessing combinations until you find one that works. Be systematic.

$$(x+1)(6x-15) \rightarrow \text{nope}$$

$$(2x+1)(3x-15) \rightarrow \text{nope}$$

$$(x-1)(6x+15) \rightarrow \text{nope}$$

$$(2x-1)(3x+15) \rightarrow \text{nope}$$

$$(x+3)(6x-5) \rightarrow \text{nope}$$

$$(2x+3)(3x-5) \rightarrow \text{yes!}$$

$$(x-3)(6x+5) \rightarrow \text{nope}$$

$$(6x+1)(x-15) \rightarrow \text{nope}$$

$$6x^2 - x - 15 = (2x+3)(3x-5)$$

$$(6x-1)(x+15) \rightarrow \text{nope}$$

$$(6x+3)(x-5) \rightarrow \text{nope}$$

$$(6x-3)(x+5) \rightarrow \text{nope}$$

Factoring By Grouping

(5)

x Factor:

$$\begin{aligned} \text{a) } x^3 + x^2 - 4x - 4 &= x^2(x+1) - 4(x+1) \\ &= (x+1)(x^2 - 4) \\ &= (x+1)(x-2)(x+2) \end{aligned}$$

$$\begin{aligned} \text{b) } 2x^3 - 12x^2 + 4x - 24 &= 2x^2(x-6) + 4(x-6) \\ &= (x-6)(2x^2 + 4) \\ &= 2(x-6)(x^2 + 2) \end{aligned}$$

Guidelines For Factoring:

- 1) Factor out any common factors
- 2) Look for and factor any special forms
- 3) Factor trinomials
- 4) Factor by grouping