A.1 – Real Numbers and their Properties

> This first section is a short review of numbers and how they work.

> First, let's think of the numbers we use to count:

$\{1, 2, 3, \ldots\}$ → These are called the natural numbers.

↳ we can add and multiply these numbers and we'll get another number in the set, but if we want to subtract, we need more numbers, ie. zero and negative numbers.

$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ → These are called the integers.

↳ we can add, subtract, and multiply these numbers and still get an answer in the set, but to divide we need more.

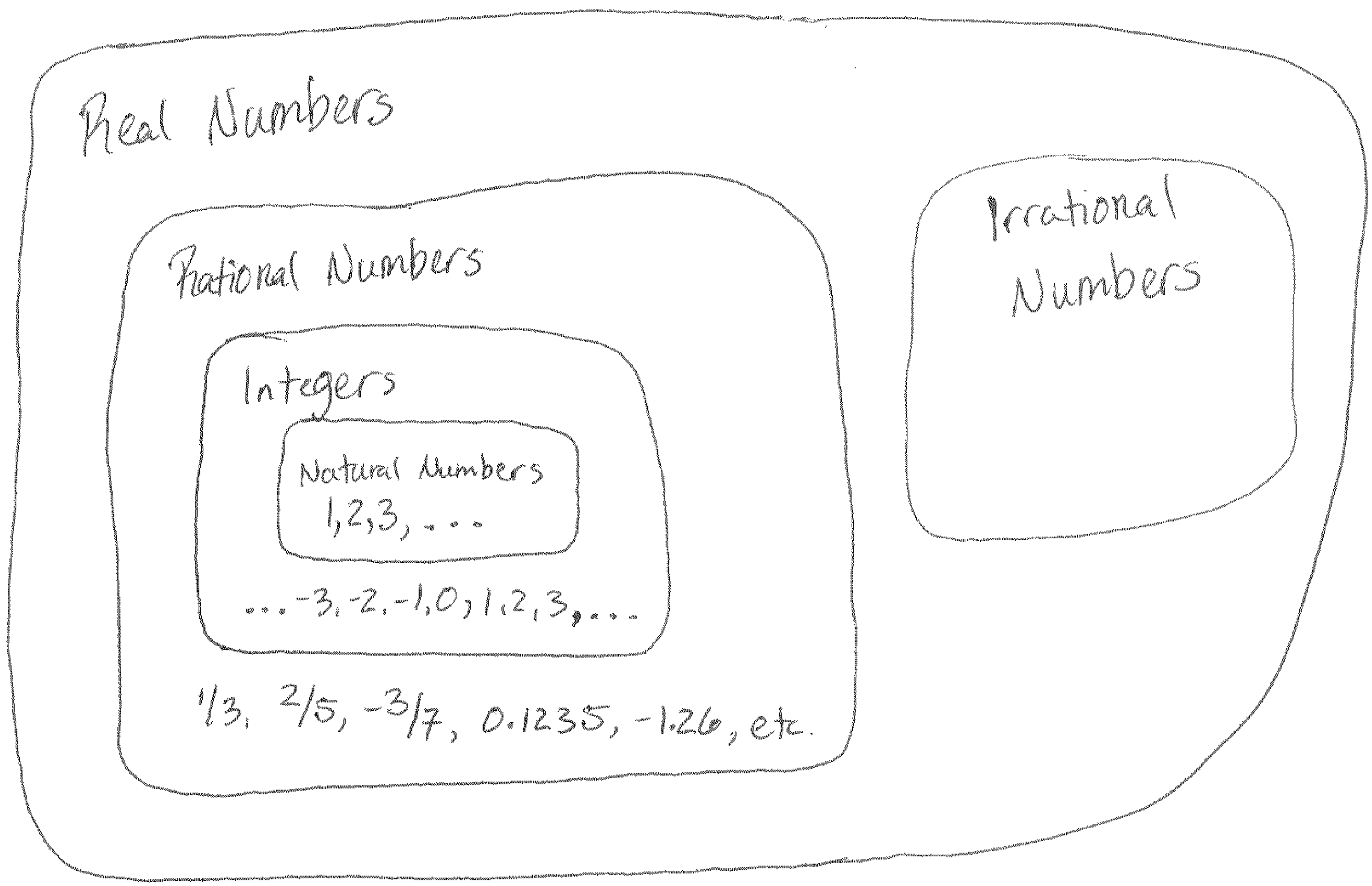
↳ a rational number can be written as $\frac{m}{n}$ where m and n are integers.

↳ This gives all terminating decimals and repeating decimals.

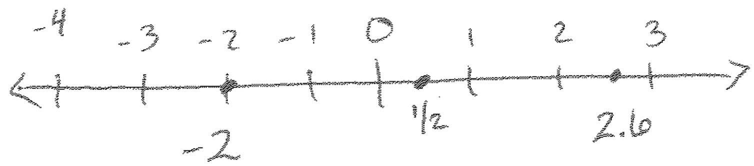
↳ Nonterminating decimals like π and $\sqrt{2}$ etc. are called irrational numbers and can't be written as the ratio of 2 integers.

↳ Putting the rational numbers together with the irrational numbers gives the <u>real numbers</u>.

↳ The real numbers are all the "normal numbers". (Basically everything except imaginary numbers)


Real Numbers
Rational Numbers
Integers
Natural Numbers
1,2,3, . . .
. . . -3, -2, -1, 0, 1, 2, 3, . . .
1/3, 2/5, -3/7, 0.1235, -1.26, etc.
Irrational Numbers

→ we visualize real numbers by plotting them on the real number line.



→ we talk about ordering of real numbers using inequalities.

Ex $2 < 3$ → 2 is less than 3

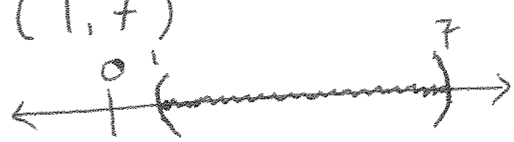
$-4 > -6$ → -6 is greater than -6

→ To talk about a range of numbers, we use intervals.

↳ To express all the numbers between 1 and 7 for example, we say

a) $1 < x < 7$

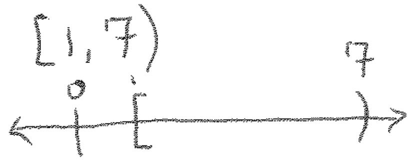
b) $(1, 7)$

c) 

↳ we can include an endpoint, say 1.

a) $1 \leq x < 7$

b) $[1, 7)$

c) 

↳ All the numbers greater than 3

a) $x > 3$. b) $(3, \infty)$. c) 

→ To talk about distances on the real number line, we use absolute values.

↳ $|x|$ → The absolute value of x

↳ This is the distance between x and zero.

EX a) $|2| = 2$ c) $|-1.35| = 1.35$

b) $|-3| = 3$ d) $|2/3| = 2/3$

↳ In practice, just get rid of the negative.

→ The rest of the section talks about the rules of how to work with real numbers. We'll just review a few of them.

element of

real numbers

Distributive Property: Let $a, b, c \in \mathbb{R}$

$$a(b+c) = ab + ac \quad \text{or} \quad (a+b)c = ac + bc$$

Properties of zero:

1) $\frac{0}{a} = 0$ (if $a \neq 0$)

2) $\frac{a}{0}$ is undefined

3) $a \cdot 0 = 0$

Fractions:

Add/Subtract: Need Common Denominator

EX a) $\frac{2}{3} + \frac{3}{4} = \frac{2\cdot 4}{3\cdot 4} + \frac{3\cdot 3}{4\cdot 3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$

b) $\frac{1}{6} - \frac{1}{8} = \frac{1\cdot 4}{6\cdot 4} - \frac{1\cdot 3}{8\cdot 3} = \frac{4}{24} - \frac{3}{24} = \frac{1}{24}$

↳ common denominator should be the least common multiple of the denominators

Multiply: Don't need common denominator

EX a) $\frac{1}{3}\cdot\frac{2}{7} = \frac{1\cdot 2}{3\cdot 7} = \frac{2}{21}$

b) $\frac{1}{3}\cdot\frac{3}{9} = \frac{1\cdot 3}{3\cdot 9} = \frac{3}{27} = \frac{1}{9}$

↳ cross cancel $\frac{1}{\cancel{3}_{1}}\cdot\frac{\cancel{3}^{1}}{9} = \frac{1}{9}$

Divide: Multiply by reciprocal

EX a) $\frac{2}{3} \div \frac{1}{4} = \frac{2}{3}\cdot\frac{4}{1} = \frac{8}{3}$

b) $\frac{\left(\frac{1}{6}\right)}{\left(\frac{2}{7}\right)} = \frac{1}{6}\cdot\frac{7}{2} = \frac{7}{12}$