9.7 - Probability

Ex Suppose you flip two coins. What is the probability that you will get two heads.

Possible outcomes:

HH → 2 heads
HT → 1 of each
TH → 1 of each
TT → 2 tails

} possible outcomes is called the sample space.

→ probability is the number of ways to get the outcome you want divided by the total number of possibilities.

$$\rightarrow P(HH) = \frac{1}{4} = 0.25$$

↳ probability is always a number between zero and one.

→ If an event E has n(E) equally likely outcomes and the sample space (S) has n(S) equally likely outcomes, then the probability of event E is

$$P(E) = \frac{n(E)}{n(S)}$$

Ex You roll two fair dice. What is the probability that the sum of the dots on the two dice is 7?

→ 6 possible numbers for each die, so a total of $6 \cdot 6 = 36$ possible outcomes for the roll

Thus $n(S) = 36$.

To get 7:

1st die	2nd die
1	6
2	5
3	4
4	3
5	2
6	1

$n(E) = 6$

$$\Rightarrow \quad P(E) = \frac{6}{36} = \frac{1}{6}$$

Ex what is the probability of being dealt 1 pair in 5 card stud?

$$\rightarrow n(S) = 52 = \frac{52!}{47!\,5!} = 2,598,960$$

$\rightarrow n(E) =$ 1 of 13 ranks $\binom{13}{1}$

2 of 4 suits $\binom{4}{2}$

3 of 12 ranks $\binom{12}{3}$

1 of 4 suits $\binom{4}{1}$

1 of 4 suits $\binom{4}{1}$

1 of 4 suits $\binom{4}{1}$

$$= \binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3$$

$$= 13 \cdot \frac{4 \cdot 3}{2} \cdot \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} \cdot 4^3$$

$$= 13 \cdot 6 \cdot 10 \cdot 11 \cdot 2 \cdot 4^3$$

$$= 1,098,240$$

$$\Rightarrow \quad P(E) = \frac{1,098,240}{2,598,960} = 0.4226$$

Ex What is the probability of rolling three 6's in a row with a fair die?

↳ Rolling a die multiple times is an independent event. That means the ~~three~~ rolls don't affect each other.

→ Probability of getting a 6 on the 1st roll is $\frac{1}{6}$

Prob on 2nd roll is $\frac{1}{6}$

Prob on 3rd roll is $\frac{1}{6}$

→ Probability on all 3 rolls is $\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{6^3} = \frac{1}{216}$

→ If A and B are independent events, the probability that both A and B will occur is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Ex A random number generator on a computer selects integers from 1 to 20. What is the probability that they are all less than or equal to 5?

→ Prob of selecting a number less than or equal to 5 is

$$P(A) = \frac{5}{20} = \frac{1}{4}$$

→ Prob of all 3 numbers less than or equal to 5 is

$$P(A) \cdot P(A) \cdot P(A) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{64}$$

Ex You open a drawer with 3 red socks, 2 yellow socks, and 4 green socks, and randomly pick.

→ what is the probability of grabbing either a red sock or a yellow sock.

→ Prob of grabbing a red sock $= \frac{3}{9} = \frac{1}{3}$

→ Prob of grabbing a yellow sock $= \frac{2}{9}$

→ Prob of grabbing a red <u>or</u> a yellow is

$$\frac{1}{3} + \frac{2}{9} = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$$

→ If A and B are mutually exclusive events (both can't occur) then

$$P(A \text{ or } B) = P(A) + P(B)$$

Ex what is the probability that you grab a matching pair of socks from the drawer above?

→ Prob of grabbing a red pair $= \dfrac{\#\text{ of ways to pick 2 of 3 red socks}}{\#\text{ of ways to pick 2 of 9 socks}}$

$$= \frac{\binom{3}{2}}{\binom{9}{2}} = \frac{\frac{3!}{2!1!}}{\frac{9!}{7!2!}} = \frac{3}{\frac{9 \cdot 8}{2}} = \frac{3}{36}$$

→ Prob. of grabbing a yellow pair $= \dfrac{\text{\# of ways to pick 2 of 2 yellows}}{\text{\# of ways to pick 2 of 9 socks}}$

$$= \frac{\binom{2}{2}}{\binom{9}{2}} = \frac{\frac{2!}{2!0!}}{36} = \frac{1}{36}$$

→ Prob. of grabbing a green pair $= \dfrac{\text{\# of ways to pick 2 of 4 greens}}{\text{\# of ways to pick 2 of 9 socks}}$

$$= \frac{\binom{4}{2}}{\binom{9}{2}} = \frac{\frac{4!}{2!2!}}{36} = \frac{\frac{4\cdot 3}{2\cdot 1}}{36} = \frac{6}{36}$$

P(matching pair) = P(red) + P(yellow) + P(green)

$$= \frac{3}{36} + \frac{1}{36} + \frac{6}{36}$$

$$= \frac{10}{36} = \boxed{\frac{5}{18}}$$