7.3 - Multivariable Linear Systems

→ In this section we move from two linear equations to multiple linear equations. We solve the system using the method of elimination like in the last section. The difficult thing is staying organized through the process.

→ Consider the following systems

row echelon form

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned} \quad \text{and} \quad \begin{aligned} x - 2y + 3z &= 9 \\ y + 3z &= 5 \\ z &= 2 \end{aligned}$$

→ The systems are equivalent, meaning they have the same solution

→ the system on the right is said to be in row echelon form.
↳ looks like triangle on the left of the = sign. & coefficient of 1st variable in each row is 1

→ The goal is to take the system on the left & make it look like the system on the right, because the system on the right is easy to solve.

Notice: $\boxed{z=2}$, so $y + 3(2) = 5 \Rightarrow y + 6 = 5 \Rightarrow \boxed{y=-1}$

so $x - 2(-1) + 3(2) = 9 \Rightarrow x + 2 + 6 = 9 \Rightarrow \boxed{x=1}$

so $(1, -1, 2)$ is the solution.

→ we change from the form on the left to the form on the right by doing <u>elementary row operations</u>. these change the form of the system but not the solution.

We can:

1) Interchange two rows

2) multiply one equation by a nonzero constant

3) Add (or subtract) a multiple of one equation to another equation & replace one of the two equations with the sum.

<u>Ex</u> Solve the following system (this was the system we saw at the beginning)

→ we want to put it in row echelon form

$$\begin{aligned} x - 2y + 3z &= 9 \quad [1] \\ -x + 3y &= -4 \quad [2] \\ 2x - 5y + 5z &= 17 \quad [3] \end{aligned}$$

Add equations [1] and [2] to get [4]

$$\begin{array}{l} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ \hline y + 3z = 5 \end{array}$$

→ [4] → replace [2] with [4]

$$\begin{aligned} x - 2y + 3z &= 9 \quad [1] \\ y + 3z &= 5 \quad [4] \\ 2x - 5y + 5z &= 17 \quad [3] \end{aligned}$$

Do 2 times [1] − [3] to get [5]

$$\begin{array}{l} 2x - 4y + 6z = 18 \\ -(2x - 5y + 5z = 17) \\ \hline y + z = 1 \end{array}$$

[5] → replace [3] with [5]

$$\begin{aligned} x - 2y + 3z &= 9 \quad [1] \\ y + 3z &= 5 \quad [4] \\ y + z &= 1 \quad [5] \end{aligned}$$

[4] − [5] to get [6]

$$\begin{array}{l} y + 3z = 5 \\ -(y + z = 1) \\ \hline 2z = 4 \end{array}$$

[6] → replace [5] with [6]

$$\begin{aligned} x - 2y + 3z &= 9 \quad [1] \\ y + 3z &= 5 \quad [4] \\ 2z &= 4 \quad [6] \end{aligned}$$

→ Divide [6] by 2

$$\begin{aligned} x - 2y + 3z &= 9 \quad [1] \\ y + 3z &= 5 \quad [4] \\ z &= 2 \quad [7] \end{aligned}$$

→ we already solved this. the solution is

$$\boxed{(1, -1, 2)}$$

Ex solve → put the system in row echelon form

$x - 3y + z = 1$ [1]
$2x - y - 2z = 2$ [2]
$x + 2y - 3z = -1$ [3]

$2 \cdot [1] - [2] = [4]$

$2x - 6y + 2z = 2$
$-(2x - y - 2z = 2)$

$-5y + 4z = 0$ → [4]

$x - 3y + z = 1$ [1]
$-5y + 4z = 0$ [4]
$x + 2y - 3z = -1$ [3]

$[1] - [3] = [5]$

$x - 3y + z = 1$
$-(x + 2y - 3z = -1)$

$-5y + 4z = 2$ → [5]

$x - 3y + z = 1$ [1]
$-5y + 4z = 0$ [4]
$-5y + 4z = 2$ [5]

$[4] - [5] = [6]$

$-5y + 4z = 0$
$-(-5y + 4z = 2)$

$0 = -2$ [6]

$x - 3y + z = 1$ [1]
$-5y + 4z = 0$ [4]
$0 = -2$ [6]

→ false statement! This means there is no solution.

we call this kind of **system** inconsistent.

Ex Solve → put the system in row echelon form

$$x + 2y - 7z = -4 \quad [1]$$
$$2x + y + z = 13 \quad [2]$$
$$3x + 9y - 36z = -33 \quad [3]$$

$2\cdot[1] - [2] = [4]$

$$\begin{array}{r} 2x + 4y - 14z = -8 \\ -(2x + y + z = 13) \\ \hline 3y - 15z = -21 \quad [4] \end{array}$$

$$x + 2y - 7z = -4 \quad [1]$$
$$3y - 15z = -21 \quad [4]$$
$$-3y + 15z = 21 \quad [5]$$

$3[1] - [3] = [5]$

$$\begin{array}{r} 3x + 6y - 21z = -12 \\ -(3x + 9y - 36z = -33) \\ \hline -3y + 15z = 21 \quad [5] \end{array}$$

$$x + 2y - 7z = -4 \quad [1]$$
$$3y - 15z = -21 \quad [4]$$
$$0 = 0 \quad [6]$$

$[4] + [5] = [6]$

$$\begin{array}{r} 3y - 15z = -21 \\ +(-3y + 15z = 21) \\ \hline 0 = 0 \end{array}$$

→ The last equation (eqn [6]) says $0z = 0$. this is true for any number. Suppose z is some number a.

$$\boxed{z = a}$$

Then $3y - 15a = -21 \Rightarrow 3y = 15a - 21 \Rightarrow \boxed{y = 5a - 7}$

so $x + 2(5a - 7) - 7a = -4 \Rightarrow x + 10a - 14 - 7a = -4$

$\Rightarrow x + 3a - 14 = -4$

$\Rightarrow \boxed{x = -3a + 10}$

So the solution is $\boxed{(-3a + 10,\ 5a - 7,\ a)}$

→ There are an infinite number of solutions. for any a, this triplet of numbers will solve the system.