7.2 → Two Variable Linear Systems

→ There is another way to solve systems of equations. It is called the **method of elimination**.

→ In the method of elimination, we add (or subtract) the equations to eliminate a variable.

Ex Solve

$$3x - 5y = 2$$
$$2x + 5y = 13$$

→ add the equations together. Since we're saying that $2x+5y$ and 13 are equal when we add $2x+5y$ on the left and 13 on the right, we're still doing the same thing to both sides.

add:

$\Rightarrow 5x = 15$

so $\boxed{x = 3}$

$3(3) - 5y = 2$

$\Rightarrow 9 - 5y = 2$

$\Rightarrow -5y = -7$

$\Rightarrow \boxed{y = 7/5}$

→ Sometimes we have to do some work before add/subtracting

Ex $\begin{aligned} x + 7y &= 12 \\ 3x - 5y &= 10 \end{aligned}$ $\xrightarrow{\cdot(-3)}$ $\begin{aligned} -3x - 21y &= -36 \\ +\ 3x - 5y &= 10 \end{aligned}$

$-26y = -26$

$\Rightarrow \boxed{y = 1}$

so $x + 7(1) = 12 \Rightarrow \boxed{x = 5}$

→ with 2 linear equations, there are 3 possible results.

---

1) Exactly 1 solution – 2 lines intersect at a point – different slopes
2) Infinitely many solutions – lines coincide – same slope & intercept
3) NO solution – lines are parallel – same slope different intercepts.

Ex Case 1:

$0.02x - 0.05y = -0.38$ $\xrightarrow{\cdot(100)}$ $2x - 5y = -38$ $\rightarrow \cdot 3$

$0.03x + 0.04y = 1.04$ $\xrightarrow{\cdot(100)}$ $3x + 4y = 104$ $\rightarrow \cdot(-2)$

$\Rightarrow \quad 6x - 15y = -114$

$-6x - 8y = -208$

$\Rightarrow \quad -23y = -322$

$\Rightarrow \boxed{y = 14}$

$3x + 4(14) = 104$

$\Rightarrow 3x + 56 = 104$

$\Rightarrow 3x = 48$

$\Rightarrow \boxed{x = 16}$

Ex: Case 2

$2x - y = 4$ $\rightarrow \cdot(2)$

$-4x + 2y = -8$

$\Rightarrow \quad 4x - 2y = 8$

$-4x + 2y = -8$

$0 = 0$ → infinitely many solutions

note $-y = -2x + 4 \Rightarrow y = 2x - 4$

$2y = 4x - 8 \Rightarrow y = 2x - 4$

↳ same line!

Ex Case 3:

$3x - y = 1 \quad \rightarrow \cdot(2)$
$-6x + 2y = 3$

$\Rightarrow \quad 6x - 2y = 2$
$-6x + 2y = 3$

$0 = 5$

↓

false statement!

no solution

note: $-y = -3x + 1 \Rightarrow y = 3x - 1$
$2y = 6x + 3 \Rightarrow y = 3x + \frac{3}{2}$
} parallel lines

Ex 1 8 oz ~~glass~~ apple juice & 1 8 oz glass orange juice together contain 185 mg vitamin C. 2 8 oz glasses apple juice & 3 8 oz glasses OJ contain 452 mg vitamin C. How much vitamin C is in a single 8 oz glass of each type?

↗ apple ↗ orange

$a + j = 185$
$2a + 3j = 452$

$\Rightarrow \quad -2a - 2j = -370$
$2a + 3j = 452$

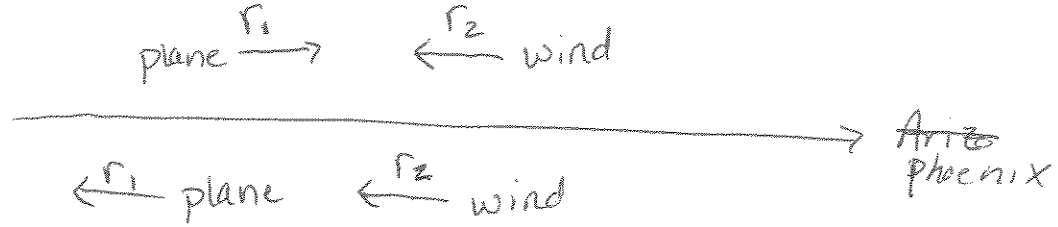
$\boxed{j = 82}$

$a + 82 = 185 \Rightarrow \boxed{a = 103}$

Ex Airplane flying into headwind travels 1800 miles between Pittsburgh and Phoenix in 3 hours & 36 minutes. It does the return trip (now with the wind) in 3 hours. What is the plane speed & wind speed assuming they both remain constant?

plane speed: r_1

wind speed: r_2



distance = (rate)(time)

From Pitt. to Phoenix:

→ 3 hours & 36 minutes is not 3.36 hours!

$$1800 = (r_1 - r_2)\left(3 + \frac{36}{60}\right)$$

From Phoenix to Pitt.

$$1800 = (r_1 + r_2)(3)$$

so $1800 = (r_1 - r_2)\left(3 + \frac{3}{5}\right) \Rightarrow 1800 = (r_1 - r_2)\left(\frac{18}{5}\right) \Rightarrow 1800 = \frac{18}{5}r_1 - \frac{18}{5}r_2$

and $1800 = (r_1 + r_2)(3) \Rightarrow 1800 = 3r_1 + 3r_2$

multiply 1st equation by 5 & second by 6

$$9000 = 18r_1 - 18r_2$$
$$10800 = 18r_1 + 18r_2$$

$$\Rightarrow 19800 = 36r_1$$
$$\Rightarrow \boxed{550 = r_1}$$

↓

plane speed

so $1800 = 3(550) + 3r_2$

$$\Rightarrow 1800 = 1650 + 3r_2$$
$$\Rightarrow 150 = 3r_2$$
$$\Rightarrow \boxed{50 = r_2}$$

↓

wind speed