7.1 - Linear and Nonlinear Systems of Equations

→ In this chapter we move from one equation to multiple connected equations

→ A solution to a set of equation is a point (or points) that makes all the equations true simultaneously.

→ One method to solve is called the method of substitution

Ex 1 Solve

$2x+y=5$
$3x-2y=4$

→ want a point (x,y) that makes both equations true.

substitution:

1 → solve for 1 variable in 1 equation: $y=5-2x$

2 → substitute into other equation: $3x-2(5-2x)=4$

3 → solve: $3x-10+4x=4 \Rightarrow 7x=14 \Rightarrow \boxed{x=2}$

4 → Solve for other variable: $y=5-2(2)$

$\Rightarrow \boxed{y=1}$

Ex 2 $x-y=-4$
$x+2y=5$

$x=y-4$ so $y-4+2y=5 \Rightarrow 3y=9 \Rightarrow \boxed{y=3}$

so $x=3-4$

$\Rightarrow \boxed{x=-1}$

→ check in original equation

→ Nonlinear equation work too.

Ex 3 $x-2y=0$

$3x-y^2=0$

so $x=2y$ then $3(2y)-y^2=0 \Rightarrow 6y-y^2=0$

$\Rightarrow y(6-y)=0$

$\Rightarrow y=0,\ y=6$

$\boxed{y=0} \Rightarrow x=2(0)$

$\Rightarrow \boxed{x=0}$

$\boxed{y=6} \Rightarrow x=2(6)$

$\Rightarrow \boxed{x=12}$

Ex 4 $x+y=4$

$x^2+y=2$

so $y=4-x$ then $x^2+4-x=2$

$\Rightarrow x^2-x+2=0$

$\Rightarrow x^2-x=-2$

$\Rightarrow x^2-x+(\frac{1}{2})^2=-2+(\frac{1}{2})^2$

$\Rightarrow (x-\frac{1}{2})^2=-2+\frac{1}{4}$

$\Rightarrow (x-\frac{1}{2})^2=\frac{-7}{4}$

$\Rightarrow x-\frac{1}{2}=\pm\sqrt{-7/4}$ → imaginary number!

→ no real solutions.

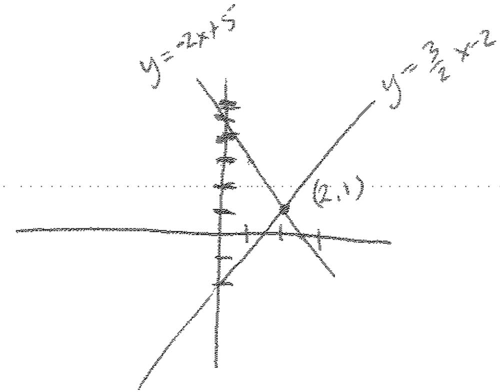
→ Graphically, real solutions are places where the graphs of the two equations intersect.

Example 1

$2x+y=5 \Rightarrow y=-2x+5$

$3x-2y=4 \Rightarrow -2y=-3x+4$

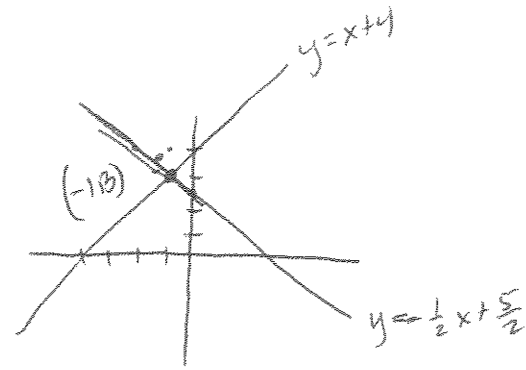
$\Rightarrow y=\frac{3}{2}x-2$



Example 2

$x-y=-4 \Rightarrow -y=-x-4 \Rightarrow y=x+4$

$x+2y=5 \Rightarrow 2y=-x+5 \Rightarrow y=-\frac{1}{2}x+5/2$

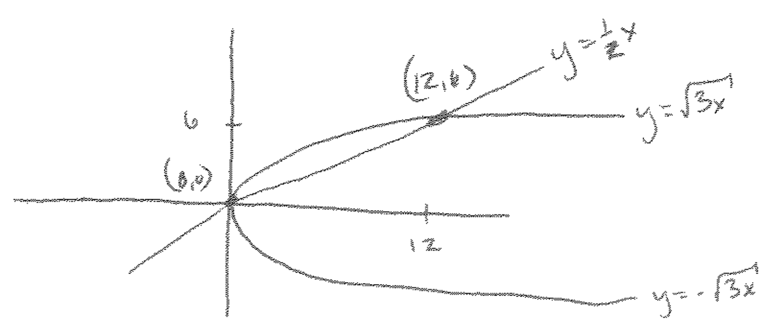


Example 4

$x-2y=0 \Rightarrow -2y=-x \Rightarrow y=\frac{1}{2}x$

$3x-y^2=0 \Rightarrow -y^2=-3x \Rightarrow y^2=3x \Rightarrow y=\pm\sqrt{3x}$

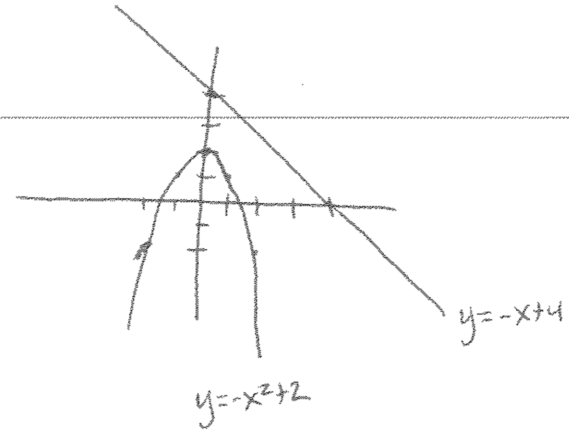
not a function, but we can still graph



Example 4

$x+y=4 \Rightarrow y=-x+4$

$x^2+y=2 \Rightarrow y=-x^2+2$



→ no intersections. Thus no real solutions.

Ex Find two numbers that add to 12 and multiply to 40.

$x+y=12$

$xy=40$

so $x=12-y$ and $(12-y)y=40$

$\Rightarrow 12y-y^2=40$

$\Rightarrow -y^2+12y=40$

$\Rightarrow y^2-12y=-40$

$\Rightarrow y^2-12y+\left(\frac{-12}{2}\right)^2=-40+\left(\frac{-12}{2}\right)^2$

$\Rightarrow (y-6)^2=-40+36$

$\Rightarrow (y-6)^2=-4$

$\Rightarrow y-6=\pm\sqrt{-4}$

$\Rightarrow y-6=\pm 2i$

$\Rightarrow y=6\pm 2i$

if $\boxed{y=6+2i}$ then $x=12-(6+2i)=12-6-2i=\boxed{6-2i}$ $x=$