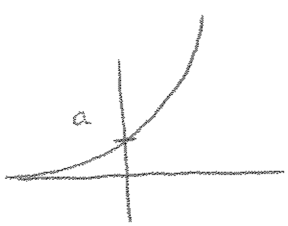
3.5 Exponential and Logarithmic Models

→ In this section we'll look at some applications of exponential and logarithmic functions. We already saw one good example of exponential growth when we talked about compound interest.

→ If you assume that the growth rate of a population is proportional to the size of the population, you get a type of equation called a differential equation. The solution of this differential equation is the exponential function

$$f(t) = ae^{bt}$$ → f is the population at time t



b controls steepness

<u>Ex</u> Fruit Fly Growth

Suppose a fruit fly population grows exponentially. If there are 100 flies after 2 days and 400 flies after 4 days, what is the model?

→ $y = ae^{bt}$ → find a and b

$100 = ae^{2b}$ and $400 = ae^{4b}$

$\Rightarrow a = \dfrac{100}{e^{2b}}$

$\Rightarrow 400 = \dfrac{100}{e^{2b}} e^{4b}$

$\Rightarrow 400 = 100e^{4b-2b}$

$\Rightarrow 4 = e^{2b}$

$\Rightarrow \ln 4 = \ln e^{2b}$

$\Rightarrow \ln 4 = 2b$

$\Rightarrow \frac{1}{2}\ln 4 = b$

$\Rightarrow \ln 4^{1/2} = b$

$\Rightarrow \boxed{\ln 2 = b}$

So then $a = \frac{100}{e^{2\ln 2}}$

$= \frac{100}{e^{\ln 2^2}}$

$= \frac{100}{e^{\ln 4}}$

$= \frac{100}{4}$

$= 25$

So the model is $\boxed{f(t) = 25e^{(\ln 2)t}}$

what was the initial population?

$f(0) = 25e^{(\ln 2)(0)}$

$= 25e^0$

$= \boxed{25 \text{ flies}}$

How many flies after 5 days

$f(5) = 25e^{(\ln 2)5}$

$= 25e^{5\ln 2}$

$= 25e^{\ln 2^5}$

$= 25e^{\ln 32}$

$= (25)(32) = \boxed{800 \text{ flies}}$

→ When scientists use carbon dating to see how old something is, they are using an exponential model.

Carbon-12 is the common carbon isotope. Carbon-14 is radioactive and has a half-life of about 5700 years. The ratio of Carbon-14 to carbon-12 molecules in organic Material is about 1 to 10^{12}. When something dies, the Carbon-12 content stays fixed but the carbon-14 decays away.

The ratio R of carbon-14 to carbon-12 present at time t can be approximated by

$$R = \frac{1}{10^{12}} e^{-t/8223}$$

Ex Scientists discover a fossil in which the ratio of carbon-14 to carbon-12 is $R = \frac{1}{10^{13}}$. How old is the fossil?

$$\overset{R \searrow}{\frac{1}{10^{13}}} = \frac{1}{10^{12}} e^{-t/8223}$$

$$\Rightarrow \frac{10^{12}}{10^{13}} = e^{-t/8223}$$

$$\Rightarrow \frac{1}{10} = e^{-t/8223}$$

$$\Rightarrow \ln\left(\frac{1}{10}\right) = \ln e^{-t/8223}$$

$$\Rightarrow \ln 1 - \ln 10 = {}^{-t}/_{8223}$$

$$\Rightarrow -\ln 10 = {}^{-t}/_{8223}$$

$$\Rightarrow t = 8223 \ln 10 \approx 18{,}934 \text{ years old}$$

→ There aren't as many examples of logarithmic equations as exponential equations. One example of a logarithmic equation is the pH scale.

→ The pH of a solution is defined as the negative log base 10 of the hydrogen ion activity

→ similar to concentration

→ $pH = -\log_{10}(x)$ where x is hydrogen ion activity.

Ex The pH of a neutral solution is 7. What is the hydrogen ion activity of a neutral solution?

$$7 = -\log_{10}(x)$$

$$\Rightarrow -7 = \log_{10}(x)$$

$$\Rightarrow 10^{-7} = 10^{\log_{10}(x)}$$

$$\Rightarrow \boxed{10^{-7} = x}$$