# 3.4 - Exponential and Logarithmic Equations



→ We have solved some simple exponential and logarithmic equations using the 1-to-1 property

Ex a) $8 = 2^x \Rightarrow 2^3 = 2^x \Rightarrow 3 = x$

b) $\ln x - \ln 5 = 0 \Rightarrow \ln x = \ln 5 \Rightarrow x = 5$

→ Our real tool for solving equations with exponentials & logarithms is the inverse properties.

Recall: $a^{\log_a x} = x$ and $\log_a a^x = x$

→ we also use the properties of logarithms

Solve:

a) $e^x = 4$ → don't want a variable in the exponent. use natural log to get it out

$\Rightarrow \ln e^x = \ln 4$

$\Rightarrow \boxed{x = \ln 4}$

b) $2(5^x) = 32$ → get rid of the coefficient out front first.

$\Rightarrow 5^x = 16$ → take logarithm of both sides

$\Rightarrow \log_5 5^x = \log_5 16$

$\Rightarrow x = \log_5 16$

→ There is a problem with this answer. WebWork, and many calculators, will only accept 2 logarithms;

→ natural log, or ln, written ln(x)

→ $\log_{10}(x)$ written logten(x)

→ There is no possible way to enter $\log_5 16$ into webwork! what do we do?

→ Recall: $\log_a u^n = n \log_a u$.

→ We can take any logarithm of $5^x$ + the x will come out front as multiplication. So lets solve with one of the logarithms that webwork can handle.

$$5^x = 16$$

$$\Rightarrow \ln(5^x) = \ln 16$$

$$\Rightarrow x \ln 5 = \ln 16$$

$$\Rightarrow \boxed{x = \frac{\ln 16}{\ln 5}}$$

$\log_5 16$ and $\dfrac{\ln 16}{\ln 5}$ are the same number!

→ In general, we have the change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \text{for any } a, b > 1$$

→ The general rule for exponential equations is to isolate the exponential and then solve with a logarithm.

Ex Solve

a) $-14 + 3e^x = 10$

$\Rightarrow 3e^x = 24$

$\Rightarrow e^x = 8$

$\Rightarrow \ln e^x = \ln 8$

$\Rightarrow \boxed{x = \ln 8}$

b) $2(3^{2t-5}) - 4 = 11$

$\Rightarrow 2(3^{2t-5}) = 14$

$\Rightarrow 3^{2t-5} = 7$

$\Rightarrow \log_{10} 3^{2t-5} = \log_{10} 7$

$\Rightarrow (2t-5)\log_{10} 3 = \log_{10} 7$

$\Rightarrow 2t-5 = \dfrac{\log_{10} 7}{\log_{10} 3}$

$\Rightarrow 2t = 5 + \dfrac{\log_{10} 7}{\log_{10} 3}$

$\Rightarrow t = \dfrac{5}{2} + \dfrac{\log_{10} 7}{\log_{10} 3} \cdot \dfrac{1}{2}$

$$\boxed{t = \frac{5}{2} + \frac{\log_{10} 7}{2\log_{10} 3}}$$

c) $\dfrac{400}{1-e^{-x}} = 350$

$\Rightarrow 400 = 350(1-e^{-x})$

$\Rightarrow \dfrac{400}{350} = 1-e^{-x}$

$\Rightarrow \dfrac{8}{7} = 1-e^{-x}$

$\Rightarrow \dfrac{1}{7} = -e^{-x}$

$\Rightarrow \dfrac{-1}{7} = e^{-x}$

$\Rightarrow \ln\left(\dfrac{-1}{7}\right) = \ln e^{-x}$

$\Rightarrow \ln\left(\dfrac{-1}{7}\right) = -x$

$\Rightarrow x = -\ln\left(\dfrac{-1}{7}\right)$

↳ can't take the logarithm of a negative number, so

$\boxed{\text{no solution}}$

→ The trick to solving logarithmic equations is to combine the logs, isolate, and then use an exponential

→ Sometimes, we may use the one-to-one property if it's easier

Ex solve

a) $\log_5 x = -3$ → use an exponential

$\Rightarrow 5^{\log_5 x} = 5^{-3}$

$\Rightarrow x = \frac{1}{5^3}$

$\Rightarrow \boxed{x = \frac{1}{125}}$

b) $\ln\sqrt{x-8} = 5$

$\Rightarrow e^{\ln\sqrt{x-8}} = e^5$

$\Rightarrow \sqrt{x-8} = e^5$

$\Rightarrow x-8 = (e^5)^2$

$\Rightarrow \boxed{x = 8 + e^{10}}$

~~c)~~ $\log_3(5x-1) = \log_3(x+7)$ → one-to-one property

$\Rightarrow 5x-1 = x+7$

$\Rightarrow 4x = 8$ → Skip!

$\Rightarrow \boxed{x=2}$

c) $\log_6(3x+14) - \log_6 5 = \log_6(2\cancel{x})$

$\Rightarrow \log_6\left(\frac{3x+14}{5}\right) = \log_6(2x)$

$\Rightarrow \frac{3x+14}{5} = 2x \Rightarrow 3x+14 = 10x \Rightarrow 14 = 7x \Rightarrow \boxed{2 = x}$

d) $\log_{10} 5x + \log_{10}(x-1) = 2$

$\Rightarrow \log_{10} 5\cancel{x}(x-1) = 2$

$\Rightarrow 10^{\log_{10} 5x(x-1)} = 10^2$

$\Rightarrow 5x^2 - 5x = 100$

$\Rightarrow x^2 - x = 20$

$\Rightarrow x^2 - x - 20 = 0$

$\Rightarrow (x-5)(x+4) = 20$

$\Rightarrow x = 5, -4$

But, if $x=-4$, we try to take the log of a negative number so $\boxed{x=5}$ is the only solution

→ Always check to see if your answer is reasonable!