3.3 - Properties of Logarithms

→ The properties of Logarithms come from the properties of exponents.

1) Product Property:
Consider the following

$$a^{\log_a(uv)} = uv = a^{\log_a u}\, a^{\log_a v} = a^{\log_a u + \log_a v}$$

so $\boxed{\log_a(uv) = \log_a u + \log_a v}$

2) Quotient Property

$$a^{\log_a\left(\frac{u}{v}\right)} = \frac{u}{v} = \frac{a^{\log_a u}}{a^{\log_a v}} = a^{\log_a u - \log_a v}$$

so $\boxed{\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v}$

3) Power Property

$$a^{n\log_a u} = \left(a^{\log_a u}\right)^n = u^n = a^{\log_a(u^n)}$$

so $\boxed{n\log_a u = \log_a u^n}$

Ex Write the following in terms of $\ln 2$ and $\ln 5$

a) $\ln 20 = \ln(4\cdot 5) = \ln 4 + \ln 5 = \ln 2^2 + \ln 5 = \boxed{2\ln 2 + \ln 5}$

b) $\ln\frac{5}{8} = \ln 5 - \ln 8 = \ln 5 - \ln 2^3 = \boxed{\ln 5 - 3\ln 2}$

Ex. Evaluate the following

a) $\log_3 \sqrt[3]{9} = \log_3 9^{1/3} = \log_3 (3^2)^{2/3} = \log_3 3^{2/3} = 2/3$

2) $\log_5 \frac{1}{125} = \log_5 1 - \log_5 125 = \log_5 1 - \log_5 5^3 = 0 - 3 = -3$

3) $\ln e^6 - \ln e^2 = 6 - 2 = 4$

or $= \ln \frac{e^6}{e^2} = \ln e^4 = 4$

4) $\log_2(-16)$ = not a real number

Ex Expand the following

1) $\log_{10} 4x^2y = \log_{10} 4 + \log_{10} x^2 + \log_{10} y$

$= \log_{10} 2^2 + 2\log_{10} x + \log_{10} y$

$= 2\log_{10} 2 + 2\log_{10} x + \log_{10} y$

2) $\ln\left(\frac{x^2-1}{x^3}\right) = \ln(x^2-1) - \ln x^3$

$= \ln\left((x-1)(x+1)\right) - 3\ln x$

$= \ln(x-1) + \ln(x+1) - 3\ln x$

3) $\log_2 \frac{\sqrt{x}\,y^4}{z^4} = \log_2 \sqrt{x} + \log_2 y^4 - \log_2 z^4$

$= \log_2 x^{1/2} + 4\log_2 y - 4\log_2 z$

$= \frac{1}{2}\log_2 x + 4\log_2 y - 4\log_2 z$

4) $\ln\sqrt{x^2(x+2)} = \ln\left(x^2(x+2)\right)^{1/2} = \frac{1}{2}\ln\left(x^2(x+2)\right)$

$= 1/2\left(\ln x^2 + \ln(x+2)\right)$

$= \frac{1}{2}\left(2\ln x + \ln(x+2)\right)$

$= \ln x + \frac{1}{2}\ln(x+2)$

Ex Condense into a Single Expression

1) $\ln y + \ln t = \ln(yt)$

2) $\ln x - 3\ln(x+1) = \ln x - \ln(x+1)^3 = \ln\frac{x}{(x+1)^3}$

3) $2\left[3\ln x - \ln(x+1) - \ln(x-1)\right] = 2\left[\ln x^3 - \left(\ln(x+1) + \ln(x-1)\right)\right]$

$= 2\left[\ln x^3 - \ln\left((x+1)(x-1)\right)\right]$

$= 2\left[\ln x^3 - \ln(x^2-1)\right]$

$= 2\left[\ln\frac{x^3}{x^2-1}\right]$

$= \ln\left(\frac{x^3}{x^2-1}\right)^2$