3.2 - Logarithmic Functions and Their Graphs

→ recall from last section that all exponential functions are one-to-one, so they all have inverses. The inverse of an exponential function is called a *logarithm*.

→ The function $f(x) = \log_a x$ "log base a of x" is defined as follows:

$$y = \log_a x \quad \text{if} \quad x = a^y$$

→ since the inverse of a^x is $\log_a x$, then if $f(x) = a^x$, $f^{-1}(x) = \log_a x$

and $f(f^{-1}(x)) = a^{\log_a x} = x$ and $f^{-1}(f(x)) = \log_a a^x = x$

→ This allows us to evaluate some logarithms (in general we need a calculator to do this)

Ex evaluate

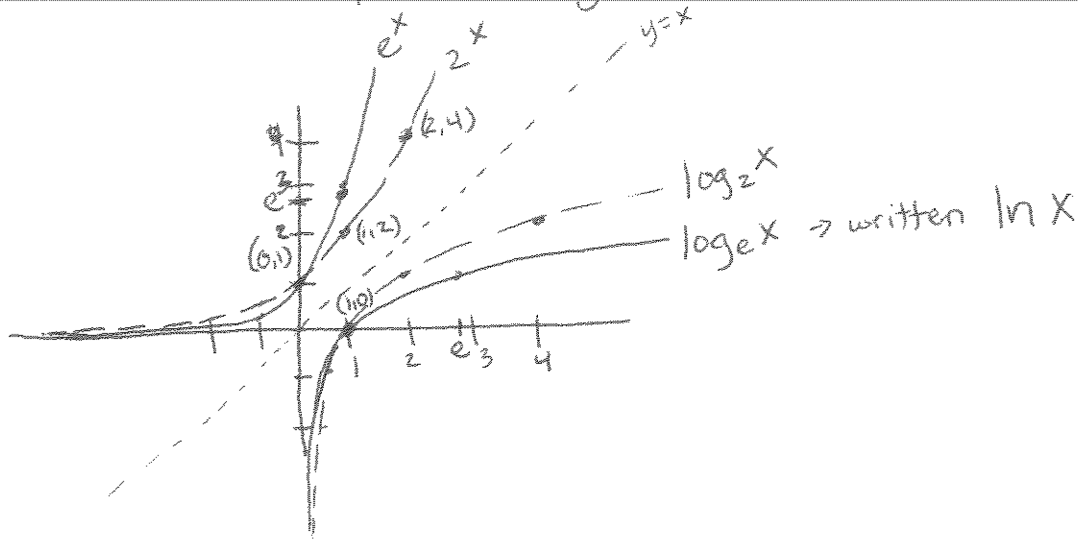
a) $\log_2 16 \rightarrow = \log_2 2^4 = 4$ write 16 as 2 to a power

b) $\log_3 27 = \log_3 3^3 = 3$

c) $\log_2 \frac{1}{8} = \log_2 \frac{1}{2^3} = \log_2 2^{-3} = -3$

a) $\log_{10} \frac{1}{100} = \log_{10} \frac{1}{10^2} = \log_{10} 10^{-2} = -2$

→ the fact that $\log_a x$ is the inverse of a^x also allows us to plot the logarithm.



Observations about the logarithm (with base $a > 1$)

→ Domain: $(0,\infty)$
→ Range: $(-\infty,\infty)$

→ Increasing on $(0,\infty)$

→ crosses x axis at $x=1$, so $\log_a 1 = 0$ for any base a

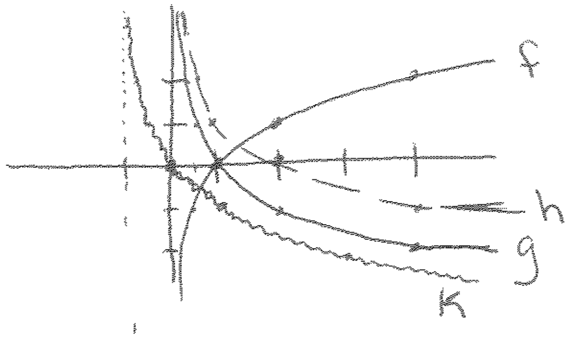
→ $\log_2 2 = 1$, $\log_e e = 1$, so $\log_a a = 1$ for any a.
→ vertical asymptote at $x=0$

Transformations of logarithms

→ Just like all functions, logarithms can be shifted, reflected, etc.

Ex: Plot $f(x) = \log_2 x$ $g(x) = -\log_2 x$ $h(x) = -\log_2 x + 1$

$k(x) = -\log_2 (x+1)$



→ Just like we called the exponential function with base e the natural exponential, we call the logarithm with base e the natural logarithm.

We write $\log_e x$ as $\ln x$ "el en x"

→ The natural logarithm is really no different than the other ones, we just write it with special notation.