2.4 - Complex Numbers

↳ we drew that picture at the beginning of the class & said that real numbers were all those things you normally think of as numbers.

↳ Now we expand to a larger set of numbers called the complex numbers. If you put the real numbers together with the complex numbers, you get the complex numbers.

↳ we define the imaginary number, i, via $i^2=-1$, or $i=\sqrt{-1}$.

→ A complex number in standard form is written

$$\underbrace{a}_{\text{real part}}+\underbrace{bi}_{\text{imaginary part}}$$

→ 2 complex numbers are equal if their real parts & imaginary parts are both equal.

→ Operations work just like polynomials or radicals.

Addition/subtraction: Combine like terms.

Let $a=2+3i$ and $b=3-i$

Ex a) $a+b = 2+3i+3-i = 5+2i$

b) $a-b = 2+3i-(3-i) = 2+3i-3+i = -1+4i$

Multiplication: Foil & replace i^2 with -1

c) Evaluate $a\cdot b$; $(2+3i)(3-i) = 6-2i+9i-3i^2 = 6+7i+3$
$= 9+7i$

Ex multiply $3-2i$ and $3+2i$

$$(3-2i)(3+2i) = 9+6i-6i-4i^2 = 9+4 = 13$$

↳ Notice that the answer is a real number. These two numbers, $3-2i$ and $3+2i$ are called complex conjugates.

→ If $a+bi$ is a complex number, $a-bi$ is called its complex conjugate conjugate. The product of complex conjugates is always real.

Ex write $\frac{3-2i}{4+i}$ as a complex number in standard form.

→ multiply numerator & denominator by complex conjugate.

$$\frac{3-2i}{4+i}\cdot\frac{4-i}{4-i} = \frac{12-3i-8i+2i^2}{16-4i+4i-i^2} = \frac{12-11i-2}{16+1} = \frac{10-11i}{17}$$

$$= \underbrace{\frac{10}{17}}_{\text{real part}} - \underbrace{\frac{11}{17}}_{\text{imaginary part}} i$$

Ex Add $\frac{i}{3-2i} + \frac{2i}{3+8i}$

→ put them in standard form so we can add their real and imaginary parts

$$\frac{i}{3-2i}\cdot\frac{3+2i}{3+2i} = \frac{3i+2i^2}{9+6i-6i-4i^2} = \frac{-2+3i}{9+4} = -\frac{2}{13}+\frac{3}{13}i$$

$$\frac{2i}{3+8i}\cdot\frac{3-8i}{3-8i} = \frac{6i-16i^2}{9-24i+24i-64i^2} = \frac{16+6i}{9+64} = \frac{16}{73}+\frac{6}{73}i$$

so we have

$$\frac{-2}{13}+\frac{3}{13}i + \frac{16}{73}+\frac{6}{73}i = \left(\frac{-2}{13}+\frac{16}{73}\right)+\left(\frac{3}{13}+\frac{6}{73}\right)i$$

$$= \frac{62}{949}+\frac{297}{949}i$$

→ when do imaginary numbers show up? For us, in taking the square root of a negative number.

Since $i^2=-1$, $\sqrt{-9} = \sqrt{9i^2} = 3i$

<u>EX</u> Simplify: $\sqrt{-27} = \sqrt{3\cdot 9\cdot i^2} = 3i\sqrt{3}$

<u>EX</u> Solve: $x^2-2x+6=0$

$\Rightarrow x^2-2x=-6$

$\Rightarrow x^2-2x+\left(-\frac{2}{2}\right)^2 = -6+\left(-\frac{2}{2}\right)^2$

$\Rightarrow (x-1)^2 = -5$

$\Rightarrow (x-1) = \pm\sqrt{-5}$

$\Rightarrow x-1 = \pm\sqrt{5}\,i$

$\Rightarrow x = +1\pm\sqrt{5}\,i$

or

$$x = \frac{2\pm\sqrt{(-2)^2-4(1)(6)}}{2(1)} = \frac{2\pm\sqrt{4-24}}{2} = \frac{2\pm\sqrt{-20}}{2} = \frac{2\pm\sqrt{5\cdot 4i^2}}{2} = \frac{2\pm 2\sqrt{5}\,i}{2} = \frac{2(1\pm\sqrt{5}\,i)}{2} = 1\pm\sqrt{5}\,i$$

→ we can now solve any quadratic equation we come across.

↳ solutions always come in complex conjugate pairs!