2.2 - Polynomial Functions of Higher Degree

↳ We know how to graph linear functions (polynomial function of degree 1) and quadratic functions (polynomial function of degree 2). In this section, we work on graphing polynomial functions of higher degree.

↳ Recall that a polynomial function has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

↳ a's are real numbers

↳ exponents are non-negative integers.

Basic Facts about polynomial functions

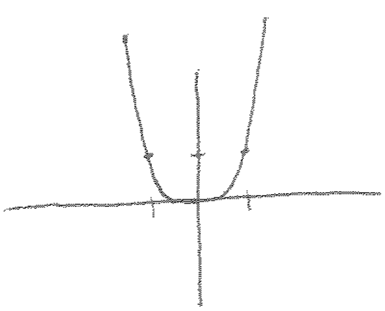
↳ Polynomial functions have domains, $(-\infty, \infty)$

↳ Graphs have no sharp corners. ($f(x) = |x|$ is not a polynomial fcn)

Monomial functions

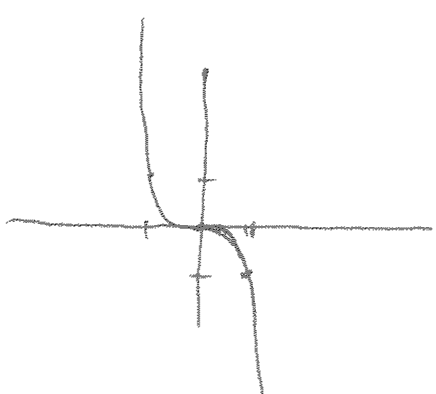
↳ Plotting monomial functions is the easiest

Ex Plot $f(x) = x^4$



→ looks like $f(x) = x^2$ but it has steeper sides & flatter bottom.

Ex Plot $f(x) = -x^5$

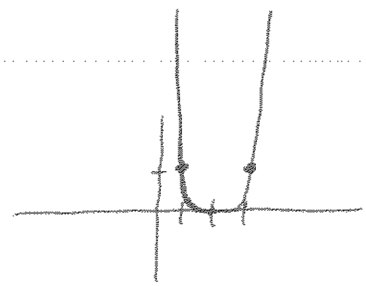


→ looks like $f(x) = -x^3$ but has steeper sides & is flattened near the x-intercept.

→ In general $f(x) = x^n$ with n even looks like $f(x) = x^2$ but the sides get steeper & it gets flatter near the vertex as n gets larger

→ In general $f(x) = x^n$ with n odd looks like $f(x) = x^3$ but the sides get steeper and it gets flatter near the intercept as n gets larger.

<u>Ex</u> Plot $f(x) = (x-2)^{10}$



→ what about functions that aren't monomials?

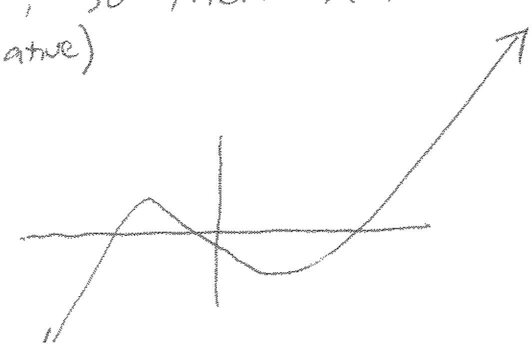
↳ one thing we can do is think about what happens when x gets very large and positive, $x \to \infty$, and as x gets very large and negative, $x \to -\infty$.

<u>Ex</u> $f(x) = 2x^3 - 3x^2 + 4x - 2$

↳ as $x \to \infty$ x^3 is way bigger than x^2, and x, so the $2x^3$ term dominates everything else. As $x \to \infty$, $2x^3 \to \infty$, so then $f(x) \to \infty$ as $x \to \infty$. (The y value gets very large)

↳ as $x \to -\infty$ the $2x^3$ term still dominates the other terms as $x \to -\infty$, $2x^3 \to -\infty$, so then $f(x) \to -\infty$. (the y value gets very large and negative)

Graph does something like this:



↳ we need to do more work to figure out the behavior closer to the origin, ~~ex~~ for example the intercepts.

~~Ex notice:~~

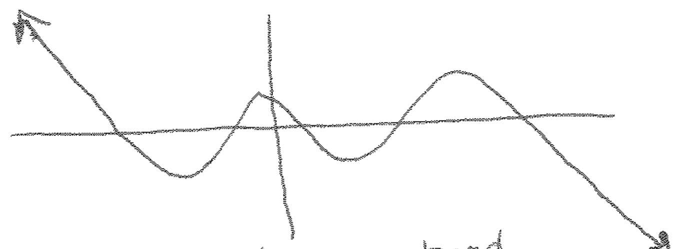
Ex $f(x) = -3x^5 + 4x^4 - 2x - 1$

→ the $-3x^5$ term dominates

→ As $x \to \infty$, $f(x) \to -\infty$

As $x \to -\infty$, $f(x) \to \infty$

↳ Graph does something like

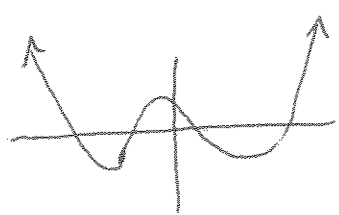


↳ Again, we need more detail to draw a good graph.

Ex $f(x) = x^4 - 3x^3 - 1$

x^4 dominates. as $x \to \infty$, $f(x) \to \infty$

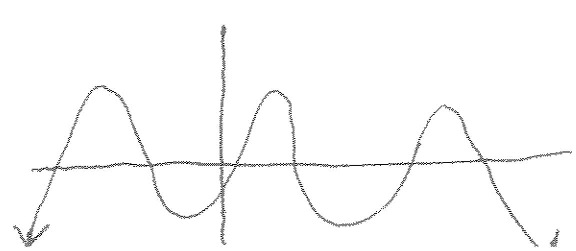
as $x \to -\infty$, $f(x) \to \infty$



Ex $f(x) = -2x^6 - 3x^5 + x^4 + 2x - 1$

$-2x^6$ dominates. as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to -\infty$



→ Notice: When the degree is odd, we start & end in different directions. When the degree is even, we start and end in the same direction.

↳ To get more detail in our graph, we find the zeros of the function.

<u>Ex</u> Plot $f(x) = -x^3 + 4x^{\cancel{2}}$.

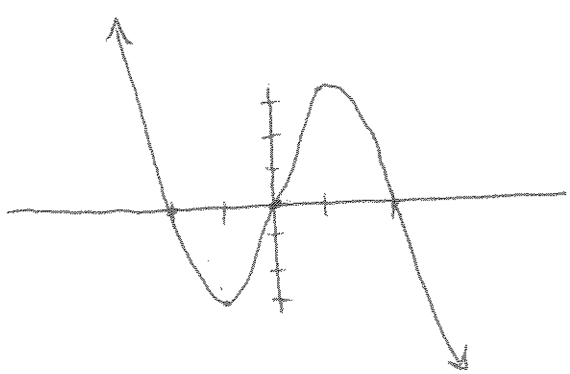
→ $f(x) \to -\infty$ as $x \to \infty$. $f(x) \to \infty$ as $x \to -\infty$

Zeros: $0 = -x^3 + 4x$
$= -x(x^2 - 4)$
$= -x(x-2)(x+2)$

Zeros a $x = 0, -2, 2$ → graph touches x-axis at these points.

↳ But does it actually cross like of $f(x) = x^3$, or does it bounce off like on $f(x) = x^2$?

↳ we should test some points.



Test: $x = -1$:
$-(-1)^3 + 4(-1)^{\cancel{2}} = 1 - 4 = -3$

$x = 1$: $-(1)^3 + 4(1)^{\cancel{2}} = -1 + 4 = 3$

Ex Plot $f(x) = x^3+4x^2+4x$

Leading coeff test:

1) as $x \to \infty$, $f(x) \to \infty$; as $x \to -\infty$, $f(x) \to -\infty$

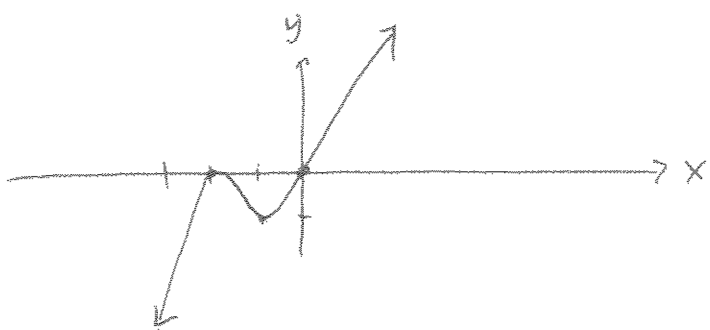
2) Find zeros: $0 = x^3+4x^2+4x$

$$= x(x^2+4x+4)$$
$$= x(x+2)(x+2)$$
$$= x(x+2)^2$$

$\Rightarrow$ zeros at $x=0$, $x=-2$

$\to$ The graph clearly has to bounce off the axis at one of the zeros, but which one?

$\to$ choose test points: $x=-1$: $(-1)^3+4(-1)^2+4(-1)$
$$= -1+4-4 = -1$$



$\hookrightarrow$ The root at $x=-2$ is called a repeated root.

Repeated roots

- If a polynomial f has a zero at $x=a$, then $(x-a)$ is a factor of f, and a is a root of f.
- If $(x-a)^k$ is a factor and k is even, the graph bounces off the axis.

↳ If $(x-a)^k$ is a factor and k is odd, the graph passes through the axis.

→ You can memorize these rules or you can plot points to see when the graph passes through the axis and when it bounces off. Plotting points is a good idea because then you also get some more information about what the graph looks like.

Ex plot: $f(x) = -x^4 + 7x^3 - 12x^2$

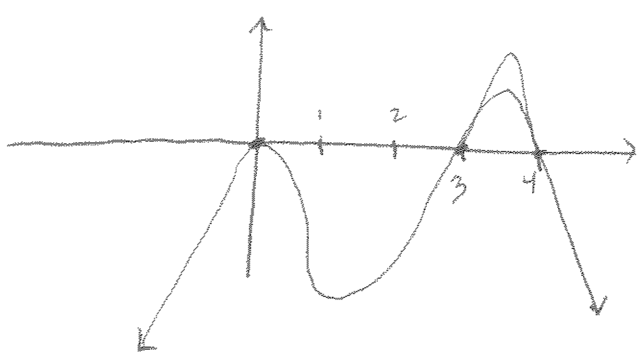
1) leading coeff. test: as $x \to \infty$, $f(x) \to -\infty$, as $x \to -\infty$ $f(x) \to -\infty$

2) zeros: $0 = -x^4 + 7x^3 - 12x^2 = -x^2(x^2+7x-12)$

$= -x^2(x-3)(x-4)$

$x=0$, $x=3$, $x=4$

↳ multiplicity 2



Test points: $x=1$: $-(1)^4 + 7(1)^3 - 12(1)^2 = -1+7-12 = -6$

$x = \frac{7}{2}$: $-(\frac{7}{2})^2(\frac{1}{2})(-\frac{1}{2})$ → plug into factored form. It's easier!

$= -\frac{49}{4} \cdot \frac{1}{2} \cdot -\frac{1}{2} = +\frac{49}{16}$