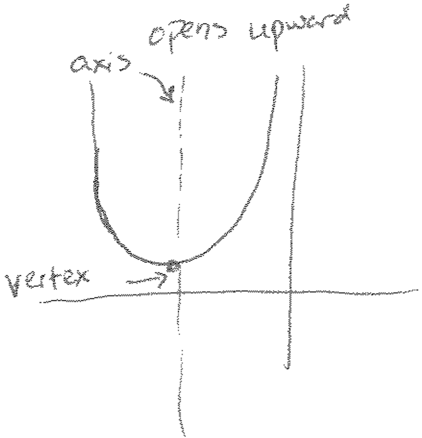
2.1 - Quadratic Functions & Models

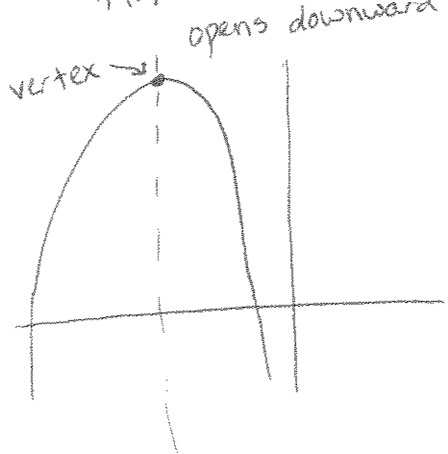
↳ We've seen that a quadratic function is a function of the form $f(x) = ax^2 + bx + c$

↳ All quadratic functions have graphs that look like parabolas.

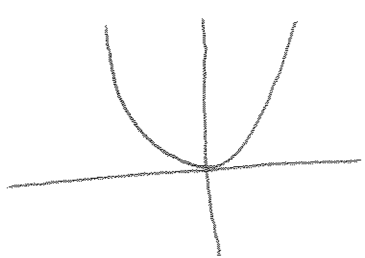
$f(x) = ax^2 + bx + c$, $a > 0$



$f(x) = ax^2 + bx + c$, $a < 0$

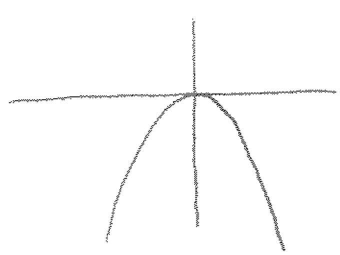


$f(x) = ax^2$, $a > 0$



vertex at origin

$f(x) = ax^2$, $a < 0$



↳ We know how to plot a parabola if the function is in the following form:

$$f(x) = a(x-h)^2 + k$$

vertical shift

vertical stretch/shrink

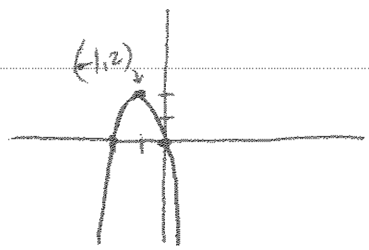
horizontal shift

②

↳ The vertex is at (h,k), a tells us ~~how~~ what the vertical shift/stretch should be, and the sign of a tells us if the parabola opens up or down.

↳ That form is called standard form.

Ex plot $f(x) = -2(x+1)^2 + 2$



? What about something like $f(x) = 3x^2 + 6x + 4$?

↳ we know it's a parabola, but what are the coordinates of the vertex?

↳ we put the function in standard form by doing something like completing the square.

$f(x) = 3x^2 + 6x + 4$

$\Rightarrow f(x) = 3\left(x^2 + 2x + \frac{4}{3}\right)$ → factor out leading coefficient

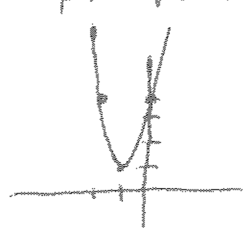
$f(x) = 3\left(x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + \frac{4}{3}\right)$ → add & subtract $\left(\frac{1}{2}\text{ coeff of } x\right)^2$

$f(x) = 3\left((x+1)^2 - 1 + \frac{4}{3}\right)$ → factor perfect square

$f(x) = 3\left((x+1)^2 + \frac{1}{3}\right)$

$f(x) = 3(x+1)^2 + 1$ → distribute 3

vertex $(-1,1)$. opens up. vertical stretch by factor of 3.



Ex Find standard form of the parabola $f(x) = -2x^2 + 8x - 7$

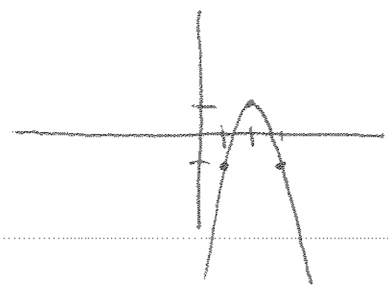
$$f(x) = -2\left(x^2 - 4x + \frac{7}{2}\right)$$

$$= -2\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + \frac{7}{2}\right)$$

$$= -2\left((x-2)^2 - 4 + \frac{7}{2}\right)$$

$$= -2\left((x-2)^2 - \frac{1}{2}\right)$$

$$= -2(x-2)^2 + 1$$



Ex Find equation in standard form of parabola with vertex at $(2,1)$ and passes through the point $(4,-3)$

$$f(x) = a(x-h)^2 + k$$

$f(x) = a(x-2)^2 + 1$ → plug in values of point for x & y. then solve for a.

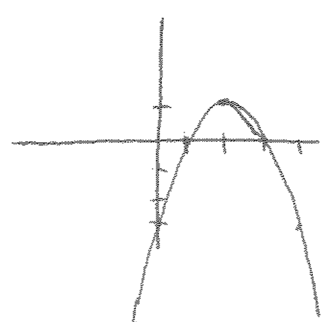
$$-3 = a(4-2)^2 + 1$$

$$\Rightarrow -3 = a(2)^2 + 1$$

$$\Rightarrow -4 = a \cdot 4$$

$$\Rightarrow -1 = a$$

so $f(x) = -(x-2)^2 + 1$



Ex Find equation of parabola with vertex $(1,1)$ that crosses the y axis at $y=2$.

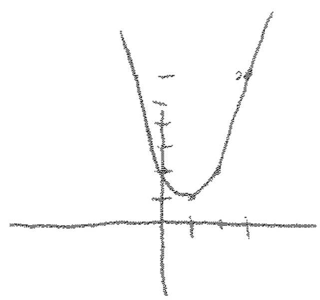
$\hookrightarrow (0,2)$

$$f(x) = a(x-1)^2+1$$

$$2 = a(0-1)^2+1$$

$$\rightarrow 2 = a+1$$

$$\Rightarrow 1 = a \qquad \Rightarrow \quad f(x) = (x-1)^2+1$$



Ex You shoot a bullet with an initial vertical velocity of 480 feet/second that leaves your gun at an initial height of 8 feet. The height of bullet at time t is given by

$$h(t) = -16t^2 + 480t + 8$$

How high does the bullet go + how long does it stay in the air?

$\hookrightarrow$ find y value of vertex of parabola.

$$h(t) = -16\left(t^2 - 30t - \frac{8}{16}\right)$$

$$= -16\left(t^2 - 30t + \left(\frac{30}{2}\right)^2 - \left(\frac{30}{2}\right)^2 - \frac{1}{2}\right)$$

$$= -16\left((t-15)^2 - 225 - \frac{1}{2}\right)$$

$$= -16\left((t-15)^2 - \frac{451}{2}\right)$$

$$\hookrightarrow h(t) = 16(t-15)^2 + 3608$$

vertex: $(15, 3608)$

$\rightarrow$ it goes 3608 feet high.

height is zero when bullet hits the ground, so solve

$$0 = -16(t-15)^2 + 3608$$

$$\Rightarrow -3608 = -16(t-15)^2$$

$$\Rightarrow \frac{3608}{16} = (t-15)^2$$

$$\Rightarrow \frac{451}{2} = (t-15)^2$$

$$\Rightarrow \pm\sqrt{\frac{451}{2}} = t-15$$

$$\Rightarrow t = 15 \pm \sqrt{\frac{451}{2}}$$

only take positive time

$$t = 15 + \sqrt{\frac{451}{2}} \approx 30.02 \text{ seconds}$$