1.8 - Combinations of Functions

↳ we can do operations on functions the same way we could do operations with polynomials; meaning we can add, subtract, multiply, and divide functions.

Here's the notation:

a) add → $(f+g)(x) = f(x)+g(x)$

b) subtract → $(f-g)(x) = f(x)-g(x)$

c) multiply → $(fg)(x) = f(x)\,g(x)$

d) divide → $\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

Ex let $f(x) = x^2+1$, $g(x) = 2x+3$

a) $(f+g)(x) = x^2+1+2x+3$
$= x^2+2x+4$

Domain: $(-\infty,\infty)$

b) $(fg)(x) = (x^2-1)(2x+3)$
$= 2x^3+3x^2-2x-3$

Domain: $(-\infty,\infty)$

c) $\left(\frac{f}{g}\right)(x) = \dfrac{x^2+1}{2x+3}$

Domain: $\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{3}{2},\infty\right)$

Ex let $f(x) = \sqrt{x+1}$, $g(x) = \sqrt{9-x^2}$

Domain of f: $x \geq -1$
$[-1,\infty)$

Domain of g: $9-x^2 \geq 0 \Rightarrow 9 \geq x^2$
$\Rightarrow -3 \leq x \leq 3$
$[-3,3]$

a) $(f-g)(x) = \sqrt{x+1} - \sqrt{9-x^2}$

Domain: $[-1,3]$

b) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$ Domain; $[-1,3)$

c) $\left(\frac{g}{f}\right)(x) = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$ Domain $(-1,3]$

↳ Other than the typical operations, there's another way to combine functions. This new way is called function composition.

Function Composition

we write the composition of function f with function g as

$(f \circ g)(x)$ which means $f(g(x))$ "f of g of x"

replace all the x's in f with the expression

Ex Let $f(x) = x+1$, $g(x) = 2x^2$

a) $(f \circ g)(x) = f(g(x)) = 2x^2 + 1$

b) $(g \circ f)(x) = g(f(x)) = 2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$

c) $(f \circ g)(2) = 2(2)^2 + 1 = 2 \cdot 4 \cdot \cancel{1} = 9$

OR $(f \circ g)(2) = f(g(2))$. Now $g(2) = 2(2)^2 = 8$ and

$f(8) = 8+1 = 9$

so $f(g(2)) = 9$

d) $(g \circ f)(2) = 2(2)^2 + 4(2) + 2 = 8+8+2 = 18$

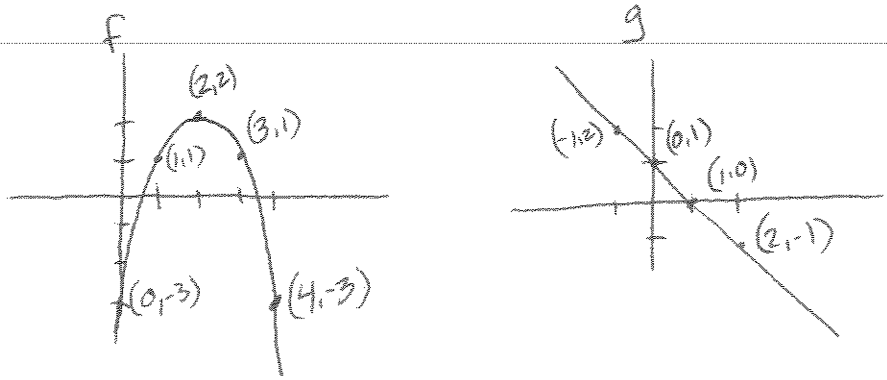
OR $(g \circ f)(2) = g(f(2))$. $f(2) = 2+1 = 3$ and

$g(3) = 2(3)^2 = 18$

so $g(f(2)) = 18$

↳ The last 2 examples show that we find an algebraic expression for the composition of 2 functions & then evaluate it or we can do it in a step-by-step way by first evaluating one function & then the next.

<u>Ex</u> Given the 2 graphs for $f(x)$ and $g(x)$, evaluate



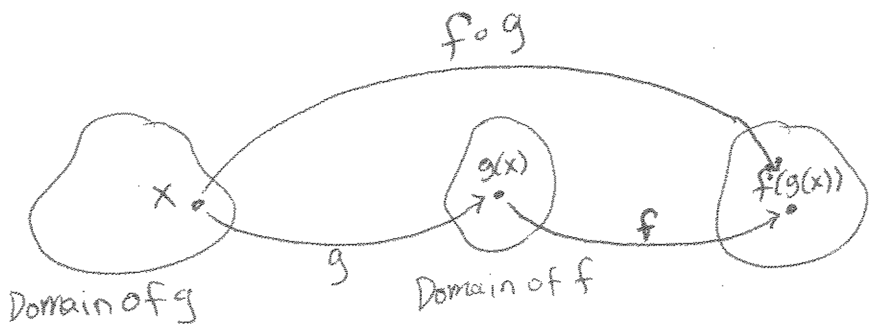
a) $f(g(1))$; $g(1)=0$ and $f(0)=-3$ so

$$f(g(1))=f(0)=-3$$

b) $(g\circ f)(2)$ $f(2)=2$ and $g(2)=-1$ so $g(f(2))=g(2)=-1$

↳ Domains get tricky with composition of functions

↳ Here's a picture of how composition works



↳ The domain of g carries through the composition.

EX Let $f(x) = x^2 - 3$, $g(x) = \sqrt{3 - x^2}$

Domain of f : $(-\infty, \infty)$; Domain of g $[-\sqrt{3}, \sqrt{3}]$ want $3 - x^2 \geq 0$

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a) $(f \circ g)(x) = f(g(x)) = (\sqrt{3-x^2})^2 \; 3 = 3 - x^2 - 3 = -x^2$

↳ The domain isn't $(-\infty, \infty)$ even though the ending expression is $-x^2$. We have to evaluate $g(x)$ & then $f(x)$. In order to evaluate $g(x)$, we need x in $[-\sqrt{3}, \sqrt{3}]$. So the domain of $(f \circ g)(x)$ is $[-\sqrt{3}, \sqrt{3}]$

↳ Sometimes we can write a function as a composition of two other functions. For example, we can write $h(x) = \frac{2}{(x+1)^3}$ as $(f \circ g)(x)$ where $f(x) = \frac{2}{x^3}$ and $g(x) = x + 1$

↳ Note: There are lots of different possibilities here. for example, $f(x) = \frac{2}{x}$ and $g(x) = (x+1)^3$ would also work.

EX Write $h(x) = \sqrt{x - 5}$ as the composition of f and g

$f(x) = \sqrt{x}$ and $g(x) = x - 5$. then $f(g(x)) = \sqrt{x-5}$