1.7 – Transformations of Functions

↳ In section 1.6, we built up a library of basic functions we called parent functions. In this section, we transform those basic functions. For all the things we graph in this section, we want to think in terms of the basic shapes of the parent functions.

↳ One way we can transform functions is to shift them vertically or horizontally

Vertical Shifts: consider the function $f(x)$...

1) vertical shift c units up: $h(x) = f(x) + c$
2) vertical shift c units down: $h(x) = f(x) - c$

→ we're just adding c to each y value.

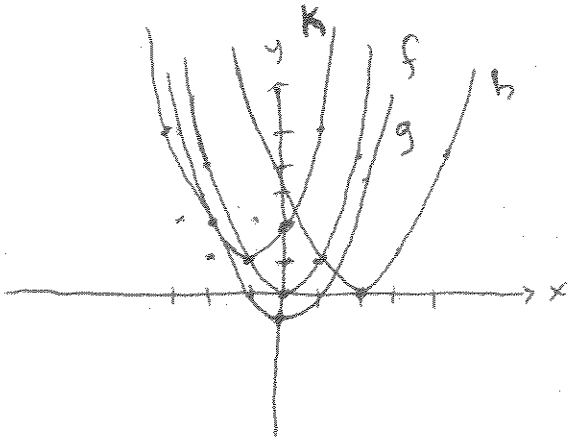
Horizontal Shifts

1) horizontal shift c units right: $h(x) = f(x-c)$
2) horizontal shift c units left: $h(x) = f(x+c)$

→ notice that it's opposite of what you would expect.

Ex use the graph of $f(x) = x^2$ to graph

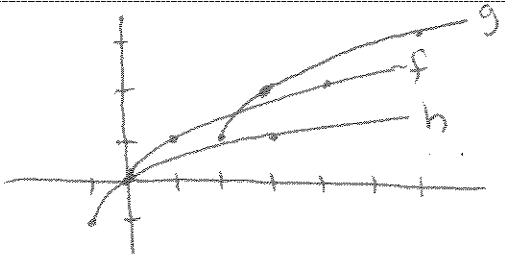
a) $g(x) = x^2 - 1$
b) $h(x) = (x-2)^2$
c) $k(x) = (x+1)^2 + 1$



Ex Use the graph of $f(x) = \sqrt{x}$ to plot

a) $g(x) = \sqrt{x-2} + 1$

b) $h(x) = \sqrt{x+1} - 1$



↳ Another way we can transform functions is by reflecting them

Reflections: consider $f(x)$

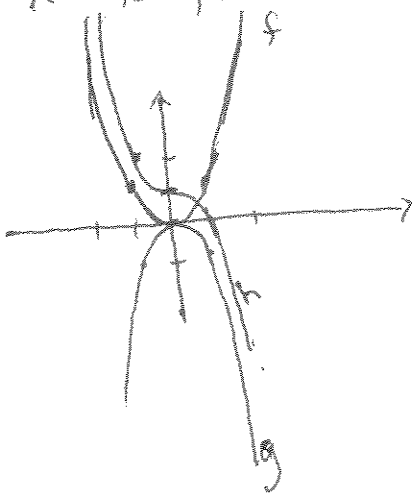
1) Reflection in x-axis: $h(x) = -f(x)$ → change sign of outputs

2) Reflection in y-axis: $h(x) = f(-x)$ → change sign of inputs

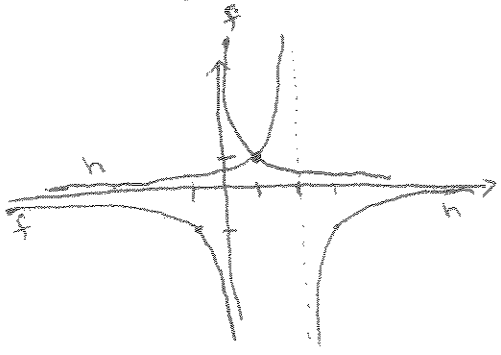
Ex use graph of $f(x) = x^3$ to plot

a) $g(x) = -x^3$

b) $h(x) = -x^3 + 1$

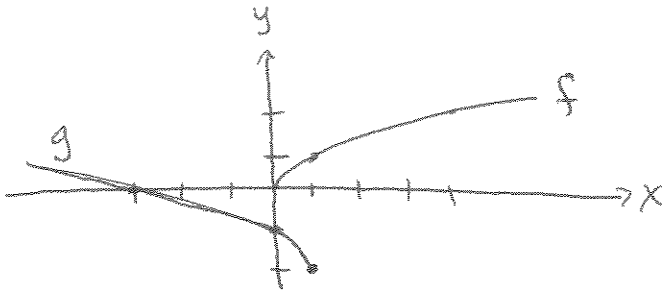


Ex Plot $h(x) = \frac{-1}{x-2}$ using $f(x) = \frac{1}{x}$



Ex Plot $g(x) = \sqrt{1-x} - 2$ using $f(x) = \sqrt{x}$

↳ This one is tricky! It's probably easiest to just plot a few points



x	g(x)
1	-2
0	-1
-3	0

↳ so its a shift down 2 & right 1 and reflected in the y-

$$g(x) = \sqrt{-(x-1)} - 2$$

- $-x$: reflect in y-axis
- $(x-1)$: 1 right
- -2: down 2

→ Any time you're confused, you can always plot points & connect the dots.

↳ there is one more type of transformation called a nongrid transformation. It is vertical & horizontal shrinks & stretches.

Vertical

$h(x) = c \cdot f(x)$ is a vertical stretch of $f(x)$ if $c > 1$ and a vertical shrink if $0 < c < 1$

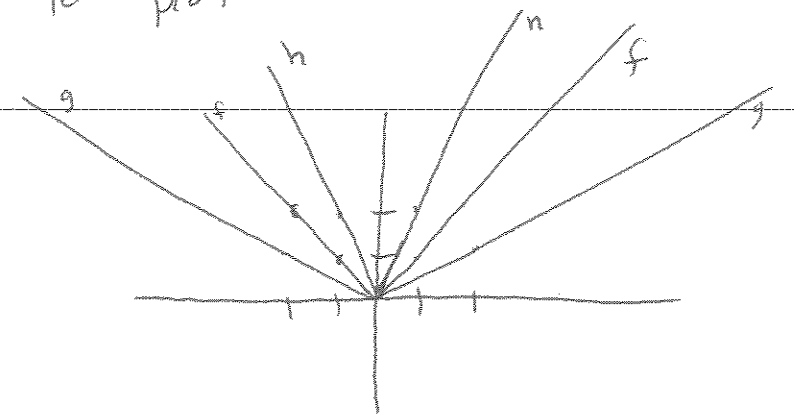
Horizontal

$h(x) = f(cx)$ is a horizontal stretch if $0 < c < 1$ and a horizontal shrink if $c > 1$.

Ex use $f(x) = |x|$ to plot

a) $g(x) = \frac{1}{2}|x|$

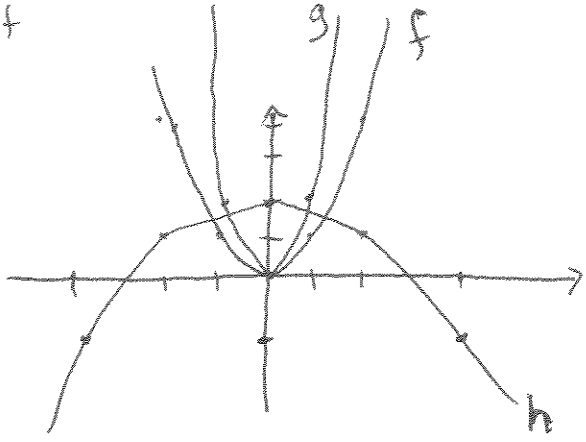
b) $h(x) = 2|x|$



Ex use $f(x) = x^2$ to plot

a) $g(x) = (2x)^2$

b) $h(x) = -(\frac{1}{2}x)^2 + 2$



↳ plotting nongrid transformations is tricky, so I usually think about this in terms of plotting points.