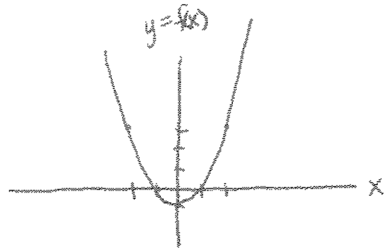
1.5 - Analyzing Graphs of Functions

↳ Graphing functions is exactly the same as graphing equations: choose input values and calculate the output values. then connect the dots.

Ex Plot $f(x) = x^2 - 1$

x	$f(x) = x^2 - 1$
-2	3
-1	0
0	-1
1	0
2	3



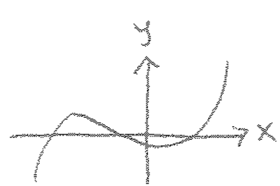
→ There was an algebraic method to see if an equation defined a function. There is also a graphical test to see if something is a function

Vertical Line Test

- A set of points is the graph of y as a function of x if a vertical line intersects the graph at at most one point

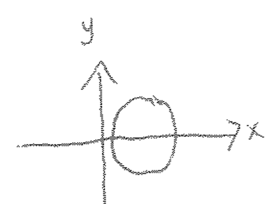
Ex

a)



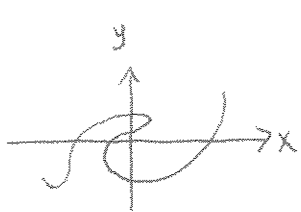
→ y is a function of x

c)



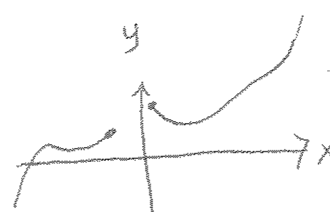
→ y is not a function of x

b)



→ y is not a function of x

d)



→ y is a function of x

↳ strange domain, but still a function

→ One aspect of a function we often use to help us graph it is the zeros of a function.

↳ These are places where $f(x) = 0$

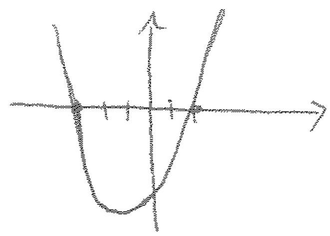
↳ same as the x-intercept

<u>Ex</u> Find the zeros

a) $f(x) = x^2 + x - 6$

$0 = x^2 + x - 6 \Rightarrow 0 = (x+3)(x-2) \Rightarrow \boxed{x = -3, 2}$

plot:



b) $f(x) = \dfrac{x^2 - 1}{x^2 + x + 1}$

$0 = \dfrac{x^2 - 1}{x^2 + x + 1}$ → only zero when numerator is zero!

$0 = x^2 - 1 \Rightarrow 0 = (x-1)(x+1) \Rightarrow \boxed{x = 1, -1}$

→ Some terminology we use when talking about functions are the words increasing, decreasing, & constant.

Technical definitions:

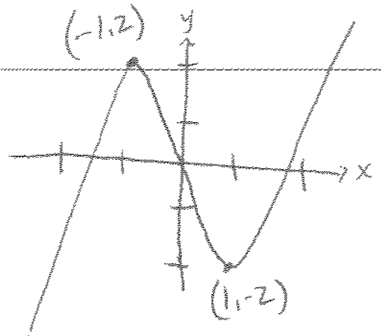
1) Function f is increasing when $x_1 < x_2$ implies $f(x_1) < f(x_2)$

2) Function f is decreasing when $x_1 < x_2$ implies $f(x_1) > f(x_2)$

3) Function f is constant when $f(x_1) = f(x_2)$ for all x_1 & x_2

→ In practice, increasing means the graph goes up and to the right, decreasing means the graph goes down and to the right, & constant means the graph is flat.

Ex $f(x) = x^3 - 3x$

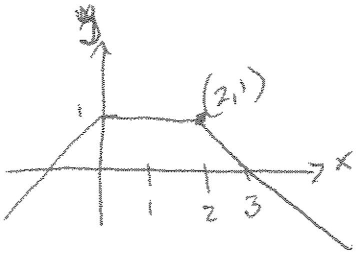


increasing: $(-\infty, -1)$ and $(1, \infty)$

or $(-\infty, -1) \cup (1, \infty)$

decreasing: $(-1, 1)$

Ex



increasing: $(-\infty, 0)$
constant: $(0, 2)$
decreasing: $(2, \infty)$

→ we talked a little about symmetry in a previous section. Some, but not all, functions have symmetry properties.

Even & Odd Functions

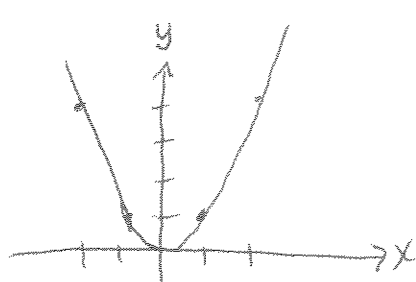
↳ A function that is symmetric with respect to the y-axis is called an even function

↳ A graph is symmetric with respect to the origin if you would fold along the x-axis & then along the y axis and all the lines will lie on top of each other.

↳ A function that is symmetric with respect to the origin is called an odd function.

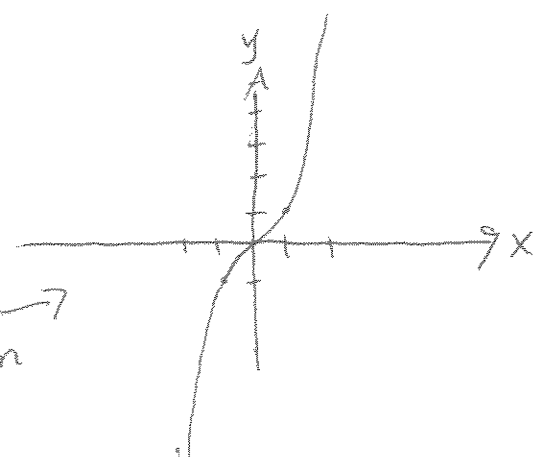
Ex $f(x) = x^2$

even function →



$f(x) = x^3$

odd function →



→ An even function has the property that

$$f(-x) = f(x) \quad \text{for any } x \text{ in the domain.}$$

→ An odd function has the property that

$$f(-x) = -f(x) \quad \text{for any } x \text{ in the domain}$$

Ex $h(x) = 3x^3 - 2x$ is odd

notice:
$$\begin{aligned} h(-x) &= 3(-x)^3 - 2(-x) \\ &= -3x^3 + 2x \\ &= -(3x^3 - 2x) \\ &= -h(x) \end{aligned}$$

→ if you plug in the negative of the x value, you get the negative of the y value

Ex $g(x) = 8x^4 - 2x^2 + 1$ is even

notice:
$$\begin{aligned} g(-x) &= 8(-x)^4 - 2(-x)^2 + 1 \\ &= 8x^4 - 2x + 1 \\ &= g(x) \end{aligned}$$

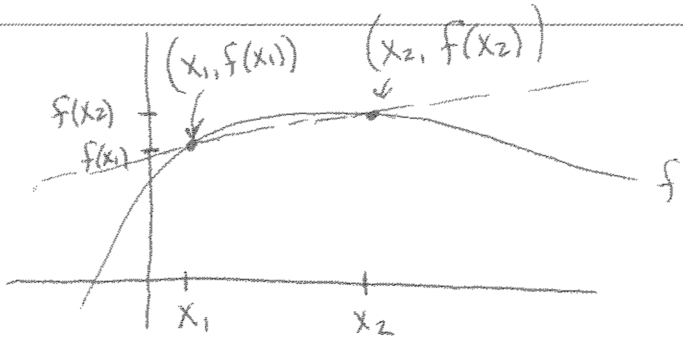
→ If you plug in the negative of the x value, you get the same y value

Average Rate of change

↳ The rate of change of a function is how much the y-value changes for each small change in the x-value. We need calculus to figure this out, but we can talk about the average rate of change.

↳ The average rate of change of a function f from x_1 to x_2 is the slope of the line connecting

$(x_1, f(x_1))$ and $(x_2, f(x_2))$



$$\text{avg rate of chage} = \text{slope} = \frac{\overbrace{f(x_2) - f(x_1)}^{\text{change in y}}}{\underbrace{x_2 - x_1}_{\text{chage in x}}}$$

Ex A car's distance (s) from a stop light is given by $s(t) = 10t^{3/2}$ where t is the time in seconds after the light has turned green. Find average speed of car from $t_1 = 0$ to $t_2 = 4$ seconds.

↳ speed $= \dfrac{\text{distance}}{\text{time}}$ → rate of change of distance

$$\text{Average speed} = \frac{s(4) - s(0)}{4 - 0} = \frac{10(4)^{3/2} - 10(0)^{3/2}}{4 - 0}$$

$$= \frac{10(\sqrt{4})^3}{4}$$

$$= \frac{10(2)^3}{4}$$

$$= \frac{10(8)}{4}$$

$$= \frac{80}{4}$$

$$= 20 \text{ feet/second}$$